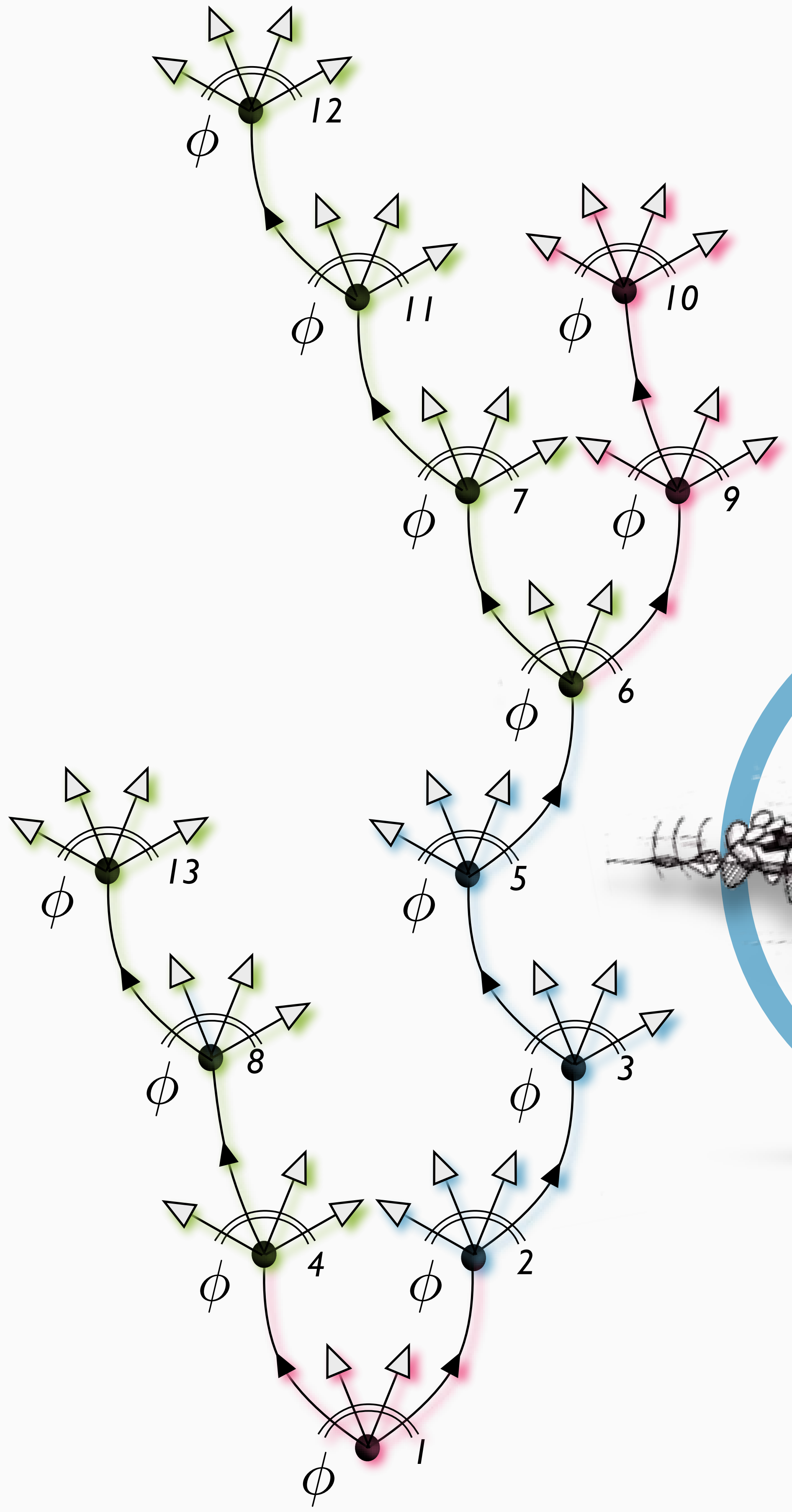
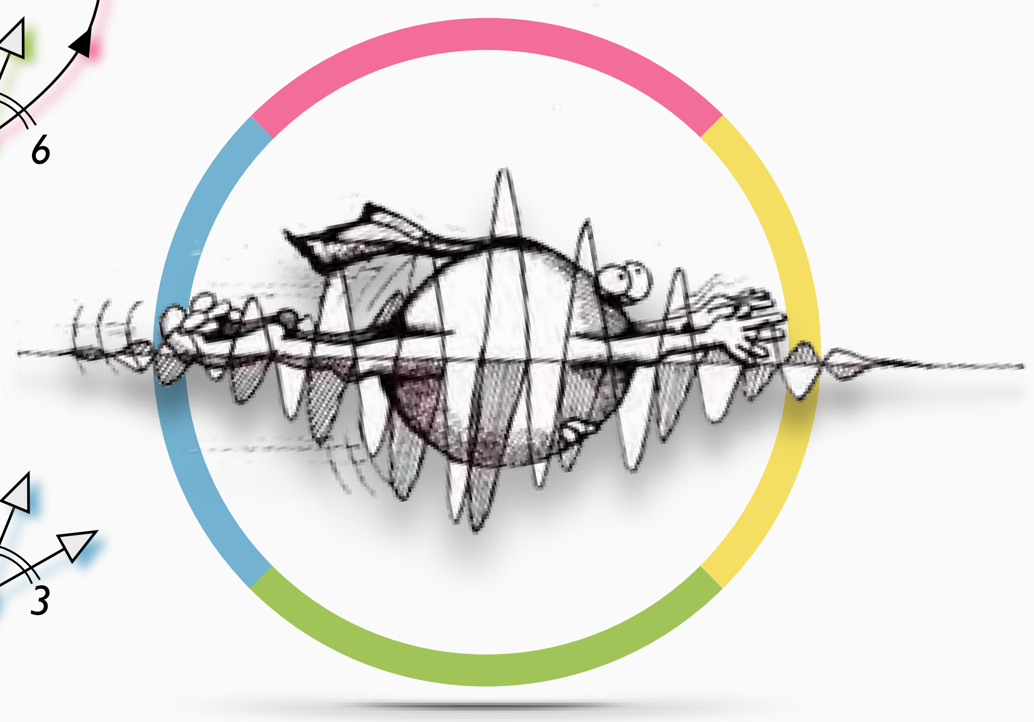




Towards Realistic Models of Quantum Phenomena

Probability in the Plato's Cave

Paweł Błasiak

Institute of Nuclear Physics
Polish Academy of Sciences, Kraków

What am I doing here?

… because of Philippe

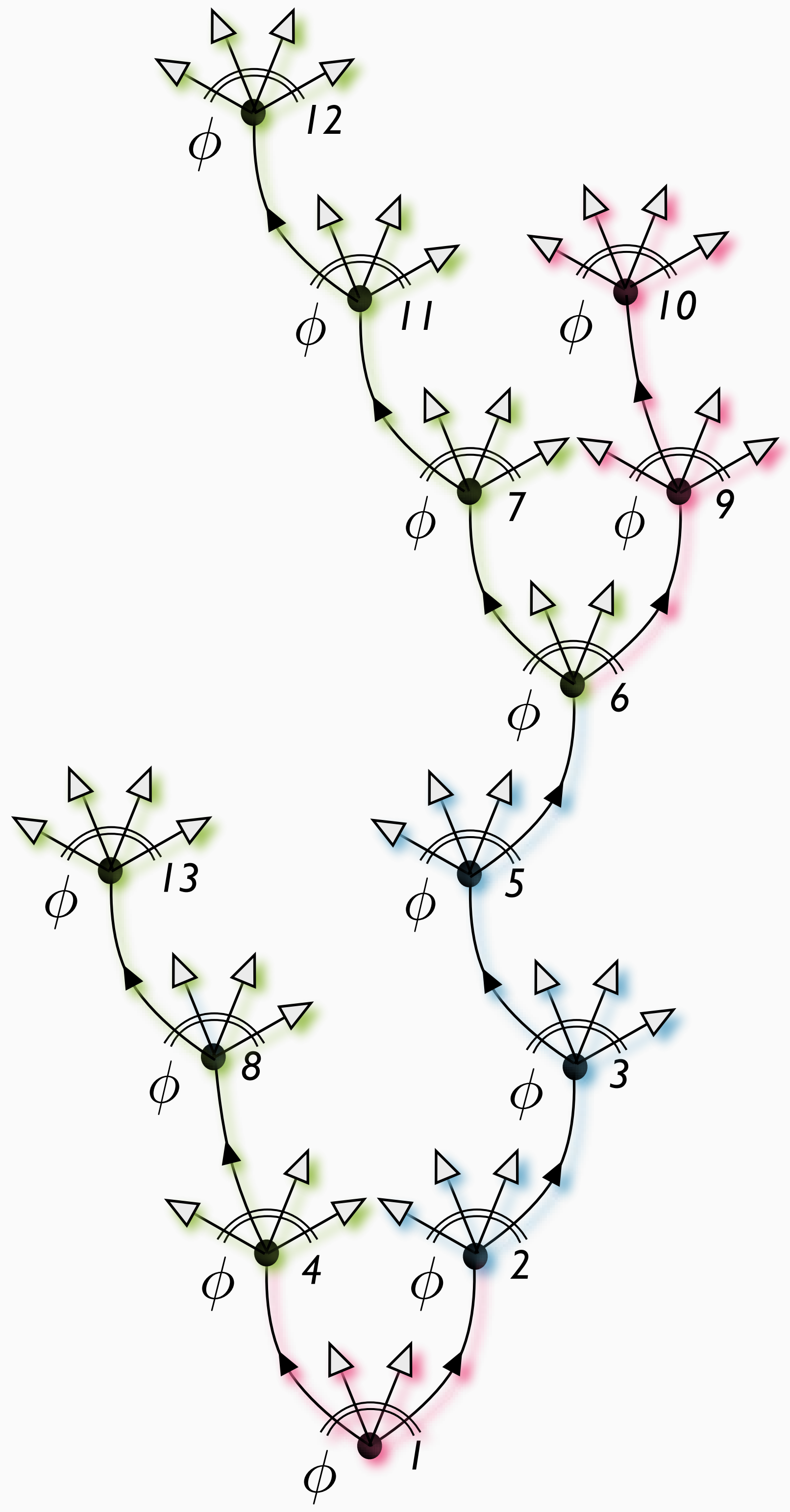
Philippe FLAJOLET
(1948 - 2011)

Séminaire Lotharingien de Combinatoire **65** (2011), Article B65c

COMBINATORIAL MODELS OF CREATION–ANNIHILATION

PAWEL BLASIAK AND PHILIPPE FLAJOLET

ABSTRACT. Quantum physics has revealed many interesting formal properties associated with the algebra of two operators, A and B, satisfying the partial commutation relation $AB - BA = 1$. This study surveys the relationships between classical combinatorial structures and the reduction to normal form of operator polynomials in such an algebra. The connection is achieved through suitable labelled graphs, or "*diagrams*", that are composed of elementary "*gates*". In this way, many normal form evaluations can be systematically obtained, thanks to models that involve set partitions, permutations, increasing trees, as well as weighted lattice paths. Extensions to q-analogues, multivariate frameworks, and urn models are also briefly discussed.



What am I doing here?

… because of Philippe

IOP Publishing — *New J. Phys.* **17** (2015) 113043 — doi:10.1088/1367-2630/17/11/113043

New Journal of Physics

The open access journal at the forefront of physics

Deutsche Physikalische Gesellschaft DPG

IOP Institute of Physics

Published in partnership with: Deutsche Physikalische Gesellschaft and the Institute of Physics

CrossMark

OPEN ACCESS

RECEIVED
12 June 2015

REVISED
21 September 2015

ACCEPTED FOR PUBLICATION
23 September 2015

PUBLISHED
18 November 2015

PAPER

Local model of a qubit in the interferometric setup

Pawel Blasiak

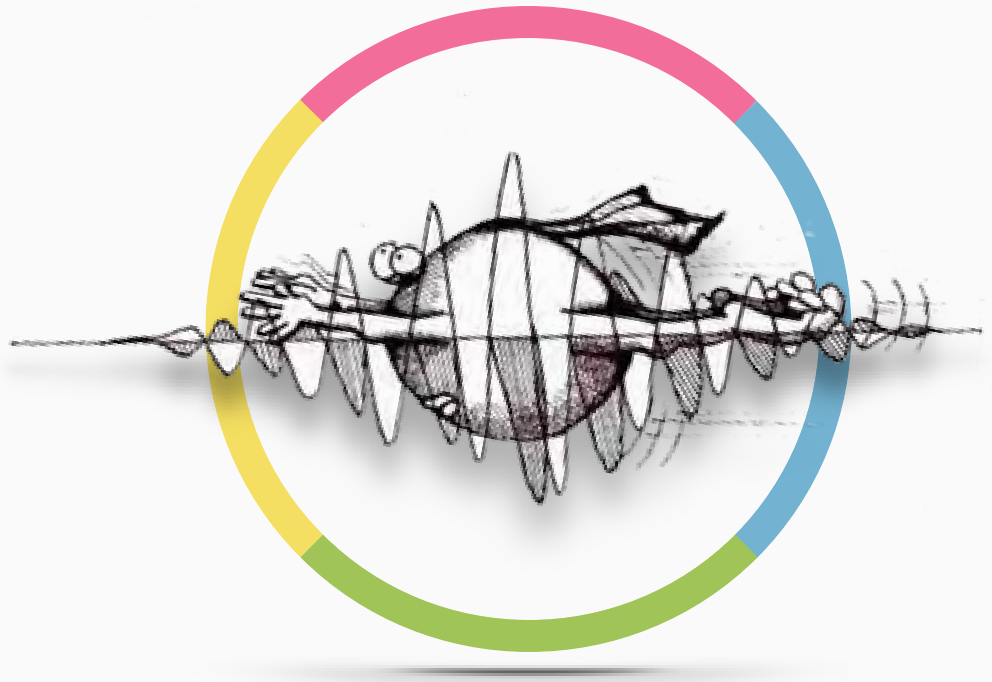
Institute of Nuclear Physics, Polish Academy of Sciences, ul. Radzikowskiego 152, 31-342 Kraków, Poland

E-mail: pawel.blasiak@ifj.edu.pl

Keywords: locality, quantum interferometry, ontological models, epistemic restrictions

Abstract

We consider a typical realization of a qubit as a single particle in two-path interferometric circuits built from phase shifters, beam splitters and detectors. This framework is often taken as a standard example illustrating various paradoxes and quantum effects, including non-locality. In this paper we show that it is possible to simulate the behaviour of such circuits in a classical manner using stochastic gates and two kinds of particles, *real* ones and *ghosts*, which interact only locally. The model has built-in limited information gain and state disturbance in measurements which are blind to *ghosts*. We demonstrate that predictions of the model are operationally indistinguishable from the quantum case of a qubit, and allegedly 'non-local' effects arise only on the epistemic level of description by the agent whose knowledge is incomplete due to the restricted means of investigating the system.



Allegory of the Cave

… are we living in a MATRIX?

Reaching further

Flammarion engraving

Quantum mechanics

Best theory we've ever had …

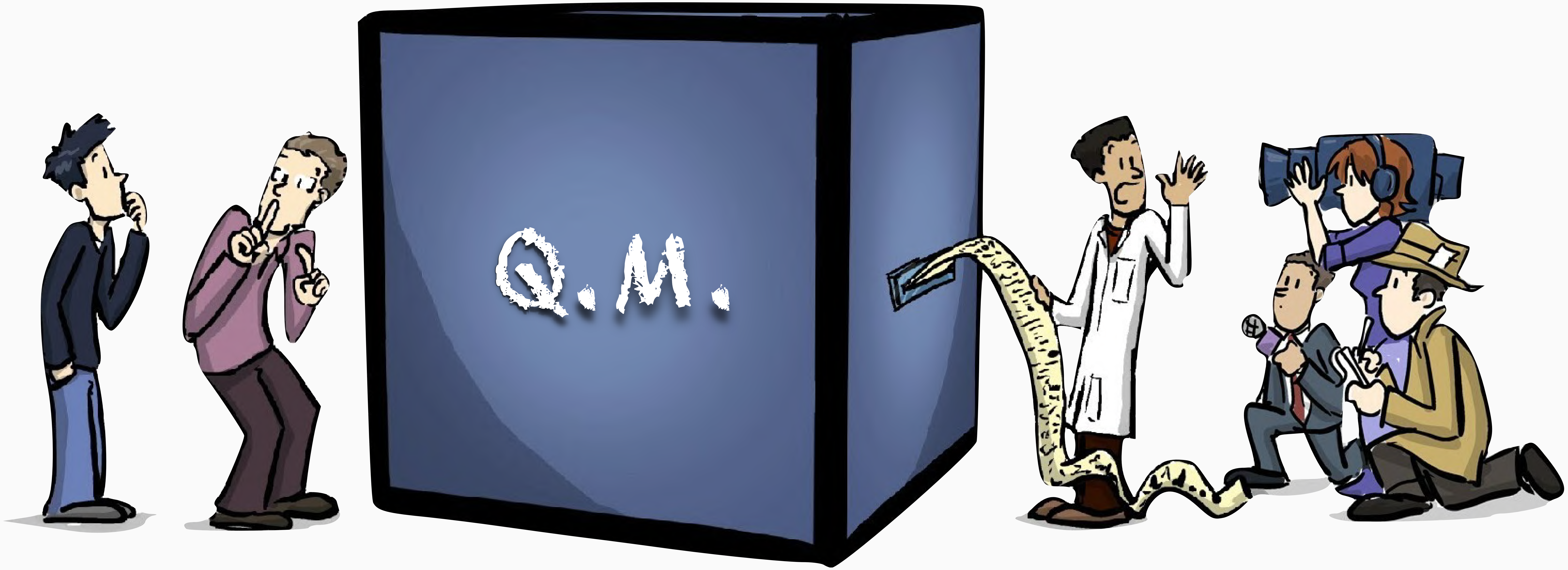
*I should begin by expressing my general attitude to present-day quantum theory, by which I mean standard non-relativistic quantum mechanics. The theory has, indeed, two powerful bodies of fact in its favour, and only one thing against it. First, in its favour are all the **marvellous agreements** that the theory has had with **every experimental result to date**. Second, and to me almost as important, it is a theory of **astonishing and profound mathematical beauty**. The one thing that can be said against it is that **it makes absolutely no sense!***

Roger Penrose
"Gravity and State Vector Reduction"
in: "Quantum Concepts in Space and Time" (1986)

Sir Roger Penrose
(1931)

Quantum mechanics

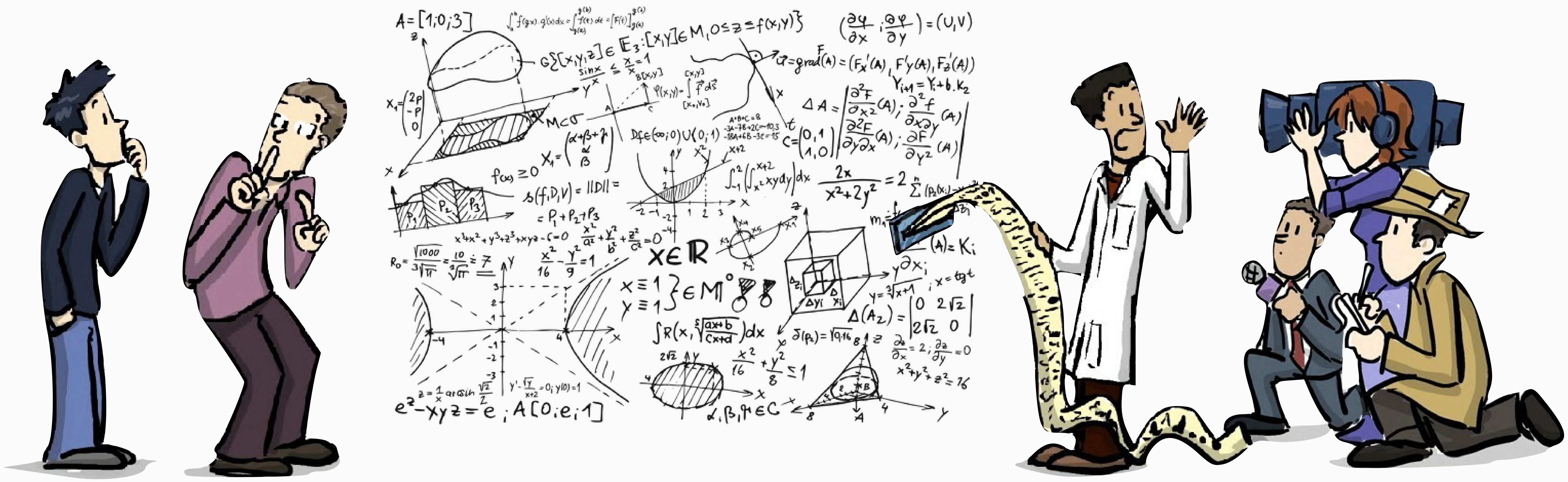
… as we have it …



Quantum mechanics

… as we have it …

Mathematical formalism ✓

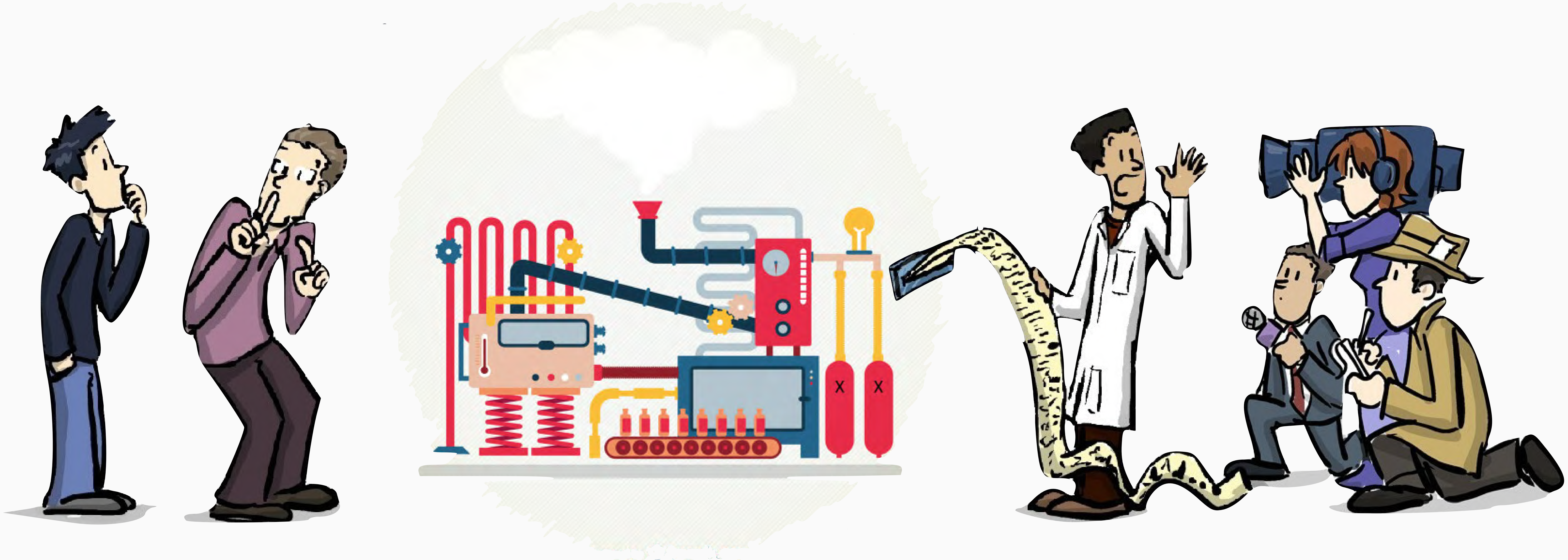


Quantum mechanics

... as we have it ...

Mathematical formalism ✓

Operational description ✓



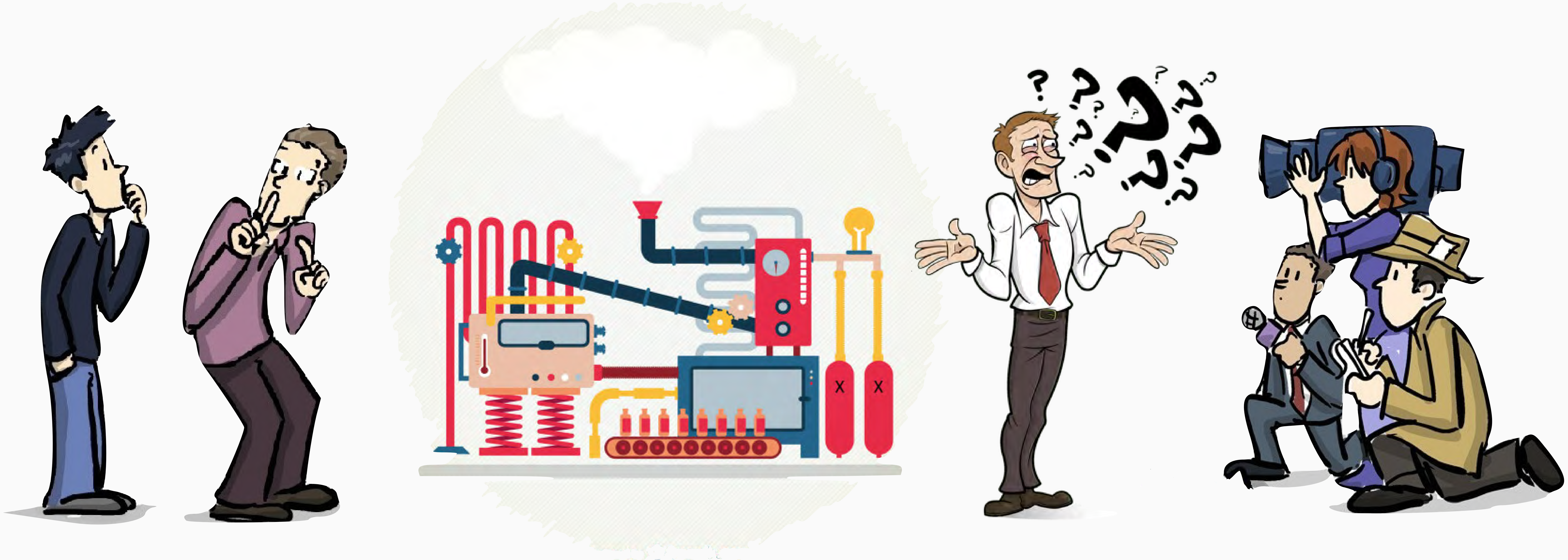
Quantum mechanics

… as we have it …

Mathematical formalism ✔

Operational description ✔

… but what is the ontology ?



Quantum mechanics

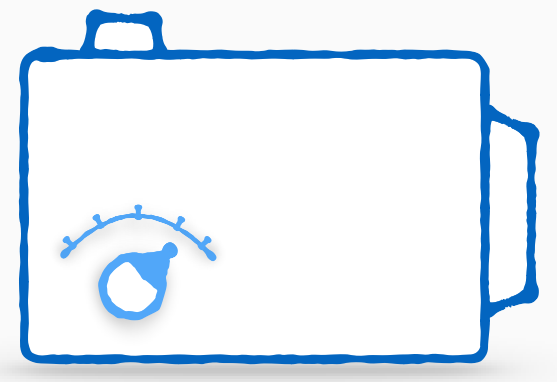
... as we have it ...

Mathematical formalism ✓

Operational description ✓

... but what is the ontology ?

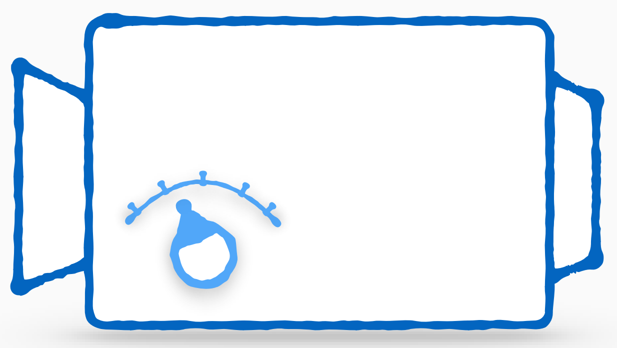
Preparation



$|\psi\rangle \in \mathcal{H}$

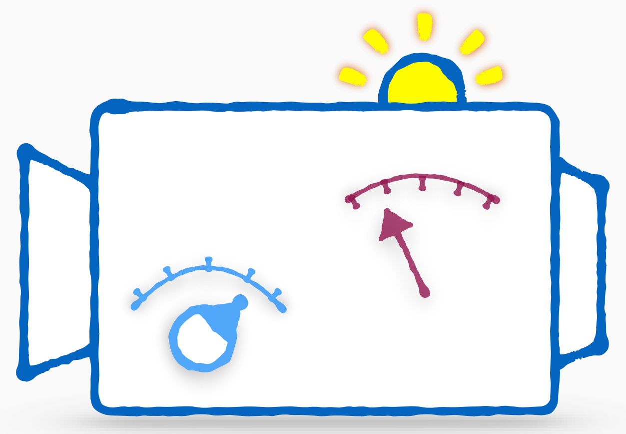
$|\psi\rangle \in \mathcal{H}_A \otimes \mathcal{H}_B$

Transformation



$|\psi\rangle \longrightarrow |\psi'\rangle = U\,|\psi\rangle$

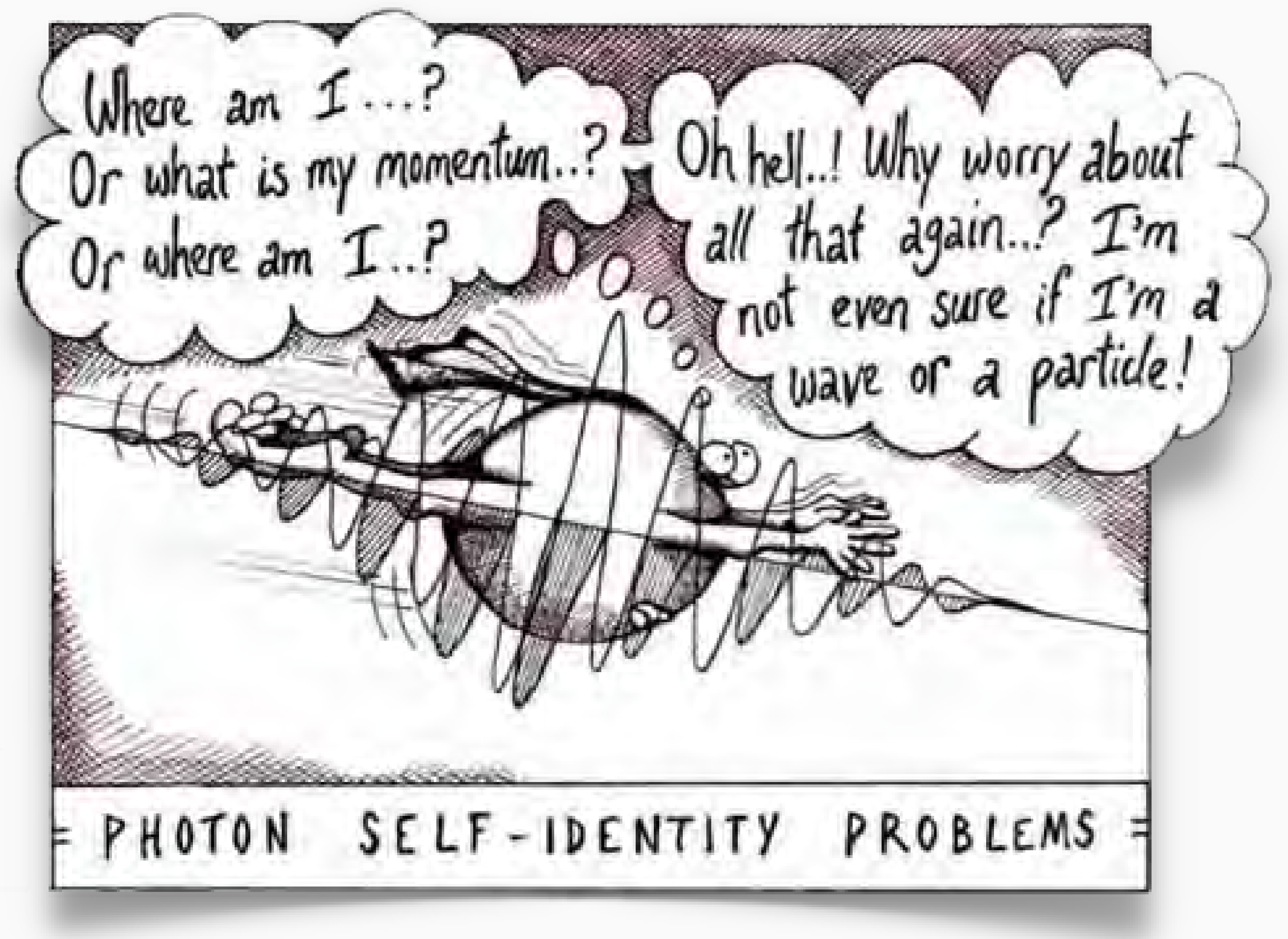
Measurement



$$\hat{A} = \sum_k a_k P_k$$

$$\mathrm{Pr}(k|\psi) = \langle\psi|\,P_k\,|\psi\rangle$$

$$|\psi\rangle \xrightarrow{k} \frac{P_k\,|\psi\rangle}{\sqrt{\langle\psi|\,P_k\,|\psi\rangle}}$$



non-locality, contextuality,
weird superposition states,
entanglement (non-local correlations),
waves or particles (both),
what is the role of observer,
no values prior to measurement,
etc...

In a strict sense, quantum theory is a ***set of rules allowing the computation of probabilities*** *for the outcomes of tests which follow specified preparations.*

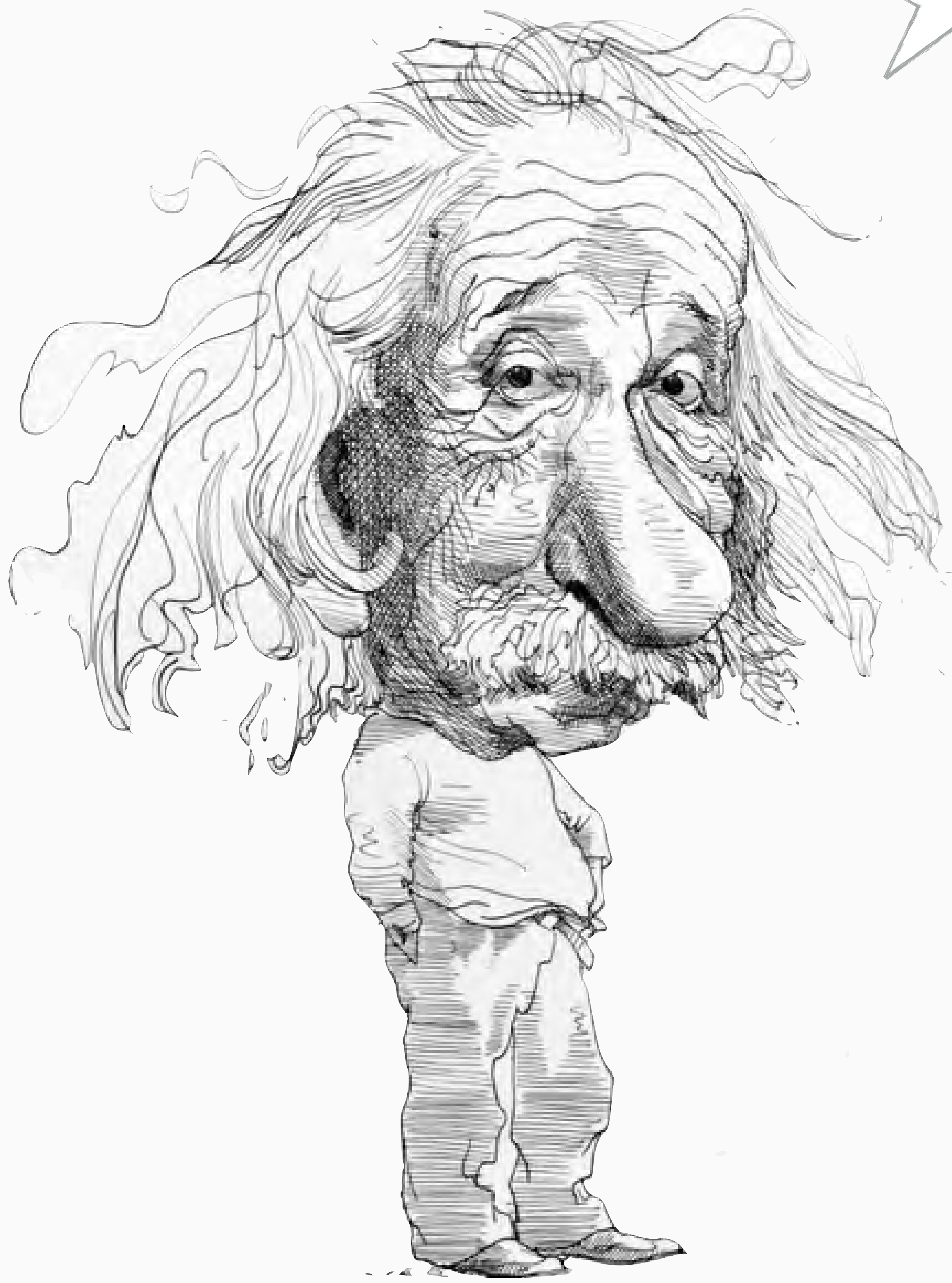
Asher Peres in Quantum Theory: Concepts and methods (1995)

Copenhagen (non-)interpretation

Einstein - Bohr debate

Niels … Get ***Real*** *!!!*

Albert … look around … it's all ***Unreal*** *!!!*



Albert EINSTEIN
(1879 - 1955)

What is the reality/story *above* the math?
Is it a '*shadow*' of something more concrete?

Niels BOHR
(1885 - 1962)

Occupation number representation

Creation—Annihilation paradigm

Occupation number representation (Fock space)

- **Hilbert space** $\mathcal{H}$ with a fixed basis:

  $|n\rangle$ - the number states

  $$|\Psi\rangle = \sum_{n=0}^{\infty} \alpha_n \, |n\rangle$$

- **Creation & annihilation** operators:

  $$a^\dagger \, |n\rangle = \sqrt{n+1} \; |n+1\rangle$$
  $$a \; |n\rangle = \sqrt{n} \; |n-1\rangle$$

- Commutator: $\boxed{[a, a^\dagger] = 1}$

- Evolution operators: $U = e^{itH(a,a^\dagger)}$

  $$|\psi_0\rangle \xrightarrow{U} |\psi_t\rangle = e^{itH(a,a^\dagger)} \, |\psi_0\rangle$$

Differential operator representation

- **Formal power series** in one variable $\mathbb{C}[[x]]$:

  x^n - polynomials

  $$F(x) = \sum_{n=0}^{\infty} f_n \, x^n$$

- **Multiplication & derivative** operators:

  $$X \, x^n = x^{n+1}$$
  $$D \, x^n = n \, x^{n-1}$$

- Commutator: $\boxed{[D, X] = 1}$

- Evolution operators: $\mathfrak{O} = e^{itH(D,X)}$

  $$F_0(x) \xrightarrow{\mathfrak{O}} F_t(x) = e^{itH(D,X)} F_0(x)$$

Heisenberg-Weyl algebra

Normal forms

Heisenberg-Weal algebra

AAU with two generators:

X - *creation (multiplication)*

D - *annihilation (derivative)*

with the relation: $DX - XD = 1$

$$\mathfrak{H} = \mathbb{C}\langle D, X\rangle \big/ [D,X] = 1$$

i.e. algebra of words with rewrite rule:

$$DX \longrightarrow XD + 1$$

- *Elements of the algebra* $\mathfrak{h} \in \mathfrak{H}$:

basis in $\mathfrak{H}$

$$\mathfrak{h} = \underbrace{\sum_{\substack{r_1,\ldots,r_k\\ s_1,\ldots s_k}} \alpha_{\substack{r_1,\ldots,r_k\\ s_1,\ldots,s_k}} X^{r_1} D^{s_1} \ldots X^{r_k} D^{s_k}}_{\text{ambiguous}} \xrightarrow{DX \longrightarrow XD+1} \mathfrak{h} = \underbrace{\sum_{r,s} \alpha_{r,s}\, X^r D^s}_{\text{unique}}$$

- *Structure constants of the algebra:* $X^p D^q X^k D^l = \sum_i \binom{q}{i}\binom{k}{i} i!\ X^{p+k-i} D^{q+l-i}$

COMBINATORIAL MODELS

(Graphs)

ALGEBRAIC STRUCTURES

(Heisenberg-Weyl algebra)

Heisenberg-Weyl algebra

Normal ordering problem

Normal ordering *and* ***normal form****:*

$$DXDDXD \xrightarrow[DX \longrightarrow XD+1]{\mathcal{N}} X^2D^4 + 4\ XD^3 + 2\ D^2$$

Change of "functional" form !!!

$$DXDDXD \xrightarrow[DX \longrightarrow XD]{:\ \ :} X^2D^4$$

commute (like numbers)

- *Operator identity:* $F(D,X) = \mathcal{N}\big(F(D,X)\big)$

  $F(D,X) \neq\ :F(D,X):$

- *Normal ordering problem*

$$\mathfrak{h} = \sum_{r,s} \alpha_{r,s}\ X^rD^s$$

$$\mathfrak{h}^n = \mathcal{N}(\mathfrak{h}^n) = \sum_{r,s} \beta_{r,s}^{(n)}\ X^rD^s$$

$$e^{z\mathfrak{h}} = \mathcal{N}(e^{z\mathfrak{h}}) = \sum_{n,r,s} \beta_{r,s}^{(n)}\ X^rD^s$$

COMBINATORIAL MODELS

(Graphs)

ALGEBRAIC STRUCTURES

(Heisenberg-Weyl algebra)

Heisenberg-Weyl algebra

Wick's theorem

Find all possible contractions:

$$
\begin{aligned}
a a^\dagger a a a^\dagger a a a^\dagger \quad &= \sum \; : \{ \text{ all contractions } \} : \\
&= \; : a a^\dagger a a a^\dagger a a a^\dagger : \\
&\quad + : a a^\dagger a a a^\dagger a \not{a} \not{a}^\dagger + a a^\dagger a a a^\dagger \not{a} a \not{a}^\dagger + a a^\dagger a \not{a} a^\dagger a a \not{a}^\dagger + \\
&\qquad a a^\dagger \not{a} a a^\dagger a a \not{a}^\dagger + \not{a} a^\dagger a a a^\dagger a a \not{a}^\dagger + a a^\dagger a \not{a} \not{a}^\dagger a a a^\dagger + \\
&\qquad a a^\dagger \not{a} a \not{a}^\dagger a a a^\dagger + \not{a} a^\dagger a a \not{a}^\dagger a a a^\dagger + \not{a} \not{a}^\dagger a a a^\dagger a a a^\dagger : \\
&\quad + : a a^\dagger a \not{a} \not{a}^\dagger a \not{a} \not{a}^\dagger + a a^\dagger \not{a} a \not{a}^\dagger a \not{a} \not{a}^\dagger + \not{a} a^\dagger a a \not{a}^\dagger a \not{a} \not{a}^\dagger + \\
&\qquad \not{a} \not{a}^\dagger a a a^\dagger a \not{a} \not{a}^\dagger + a a^\dagger a \not{a} \not{a}^\dagger \not{a} a \not{a}^\dagger + a a^\dagger \not{a} a \not{a}^\dagger \not{a} a \not{a}^\dagger + \\
&\qquad \not{a} a^\dagger a a \not{a}^\dagger \not{a} a \not{a}^\dagger + \not{a} \not{a}^\dagger a a a^\dagger \not{a} a \not{a}^\dagger + a a^\dagger \not{a} \not{a} \not{a}^\dagger a a \not{a}^\dagger + \\
&\qquad \not{a} a^\dagger a \not{a} \not{a}^\dagger a a \not{a}^\dagger + \not{a} \not{a}^\dagger a \not{a} a^\dagger a a \not{a}^\dagger + a a^\dagger \not{a} \not{a} \not{a}^\dagger a a \not{a}^\dagger + \\
&\qquad \not{a} a^\dagger \not{a} a \not{a}^\dagger a a \not{a}^\dagger + \not{a} \not{a}^\dagger \not{a} a a^\dagger a a \not{a}^\dagger + \not{a} a^\dagger a \not{a} \not{a}^\dagger a a \not{a}^\dagger + \\
&\qquad \not{a} a^\dagger \not{a} a \not{a}^\dagger a a \not{a}^\dagger + \not{a} \not{a}^\dagger a \not{a} \not{a}^\dagger a a a^\dagger + \not{a} \not{a}^\dagger \not{a} a \not{a}^\dagger a a a^\dagger : \\
&\quad + : \not{a} \not{a}^\dagger a \not{a} \not{a}^\dagger a \not{a} \not{a}^\dagger + \not{a} \not{a}^\dagger \not{a} a \not{a}^\dagger a \not{a} \not{a}^\dagger + \not{a} \not{a}^\dagger a \not{a} \not{a}^\dagger \not{a} a \not{a}^\dagger + \\
&\qquad \not{a} \not{a}^\dagger \not{a} a \not{a}^\dagger \not{a} a \not{a}^\dagger + \not{a} \not{a}^\dagger \not{a} \not{a} \not{a}^\dagger a a \not{a}^\dagger + \not{a} \not{a}^\dagger \not{a} \not{a} \not{a}^\dagger a a \not{a}^\dagger : \\
&= \; a^{\dagger\,3} a^5 + 9\, a^{\dagger\,2} a^4 + 18\, a^\dagger a^3 + 6\, a^2
\end{aligned}
$$

Integers → ***Combinatorics ...***

GOOD *for computer algebra ...*

Problematic *for infinite series ...*

!! NOT CONSTRUCTIVE !!
for analytical calculations ...

COMBINATORIAL MODELS

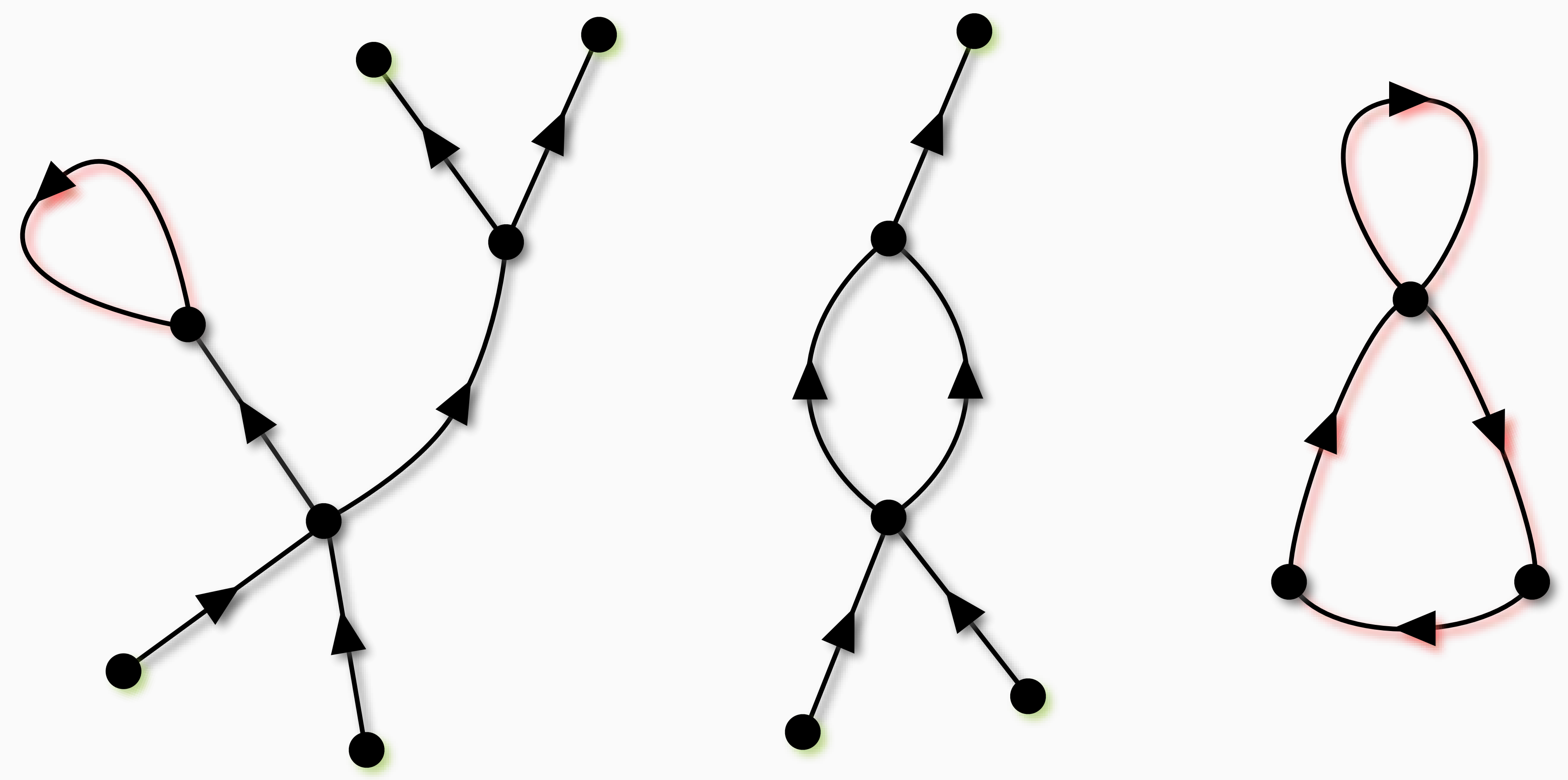
(Graphs)

ALGEBRAIC STRUCTURES

(Heisenberg-Weyl algebra)

*A **directed graph** is a collection of **edges** E and **vertices** V together with two mappings $h, t : E \longrightarrow V$ prescribing how the **head** and **tail** of each edge is attached to vertices.*

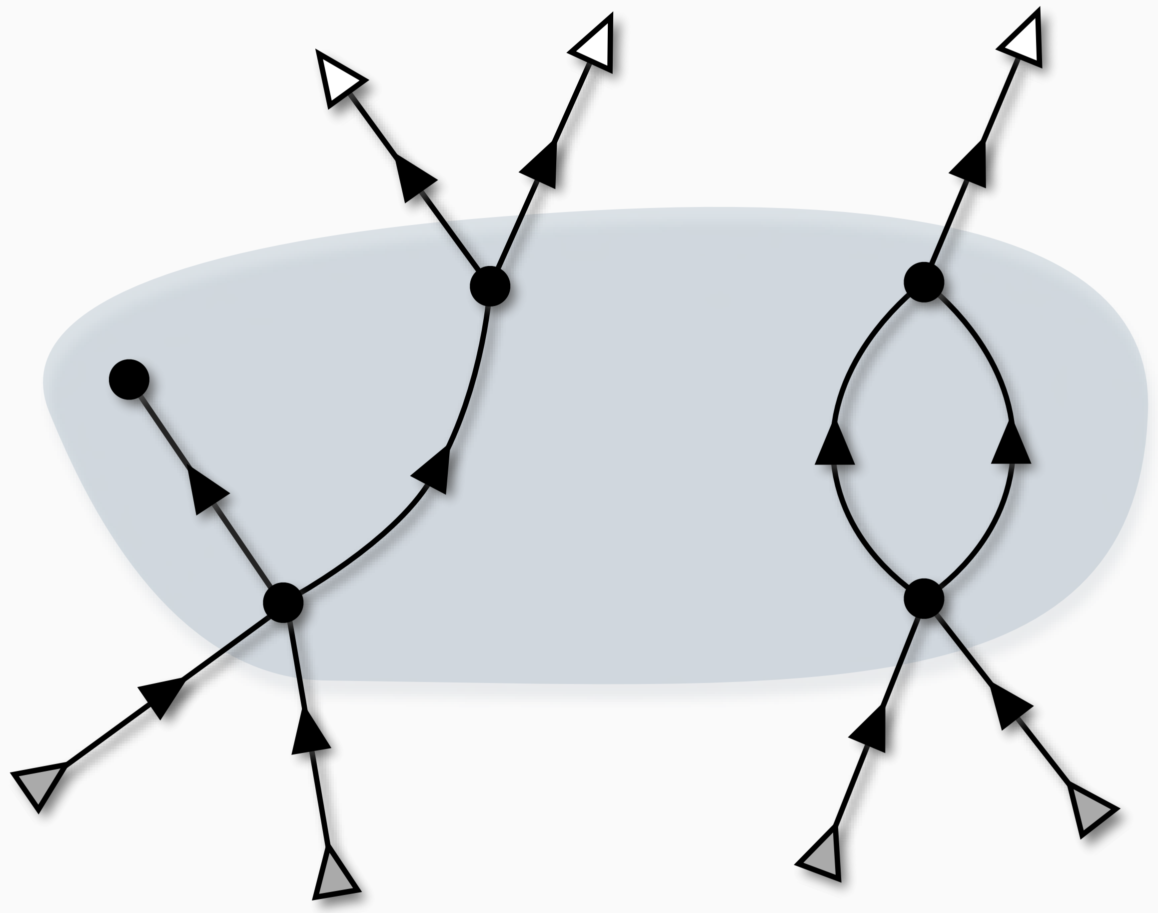
Example:



- *We shall consider **classes of graphs** up to isomorphism, i.e. simply pictures.*
- *Take **plane graphs** (not planar), i.e. lines going in/out of a vertex are **ordered** !!!*
- *Following a **cycle** in a graph one ends at the starting point.*

Definition:

Combinatorial class of ***Heisenberg - Weyl graphs*** *consists of* ***plane directed*** *graphs* Γ *which* ***do not have cycles*** *and may be* ***partially-defined****.*



Γ^+ *outgoing edges*

Γ^0 *inner edges*

Γ^- *ingoing edges*

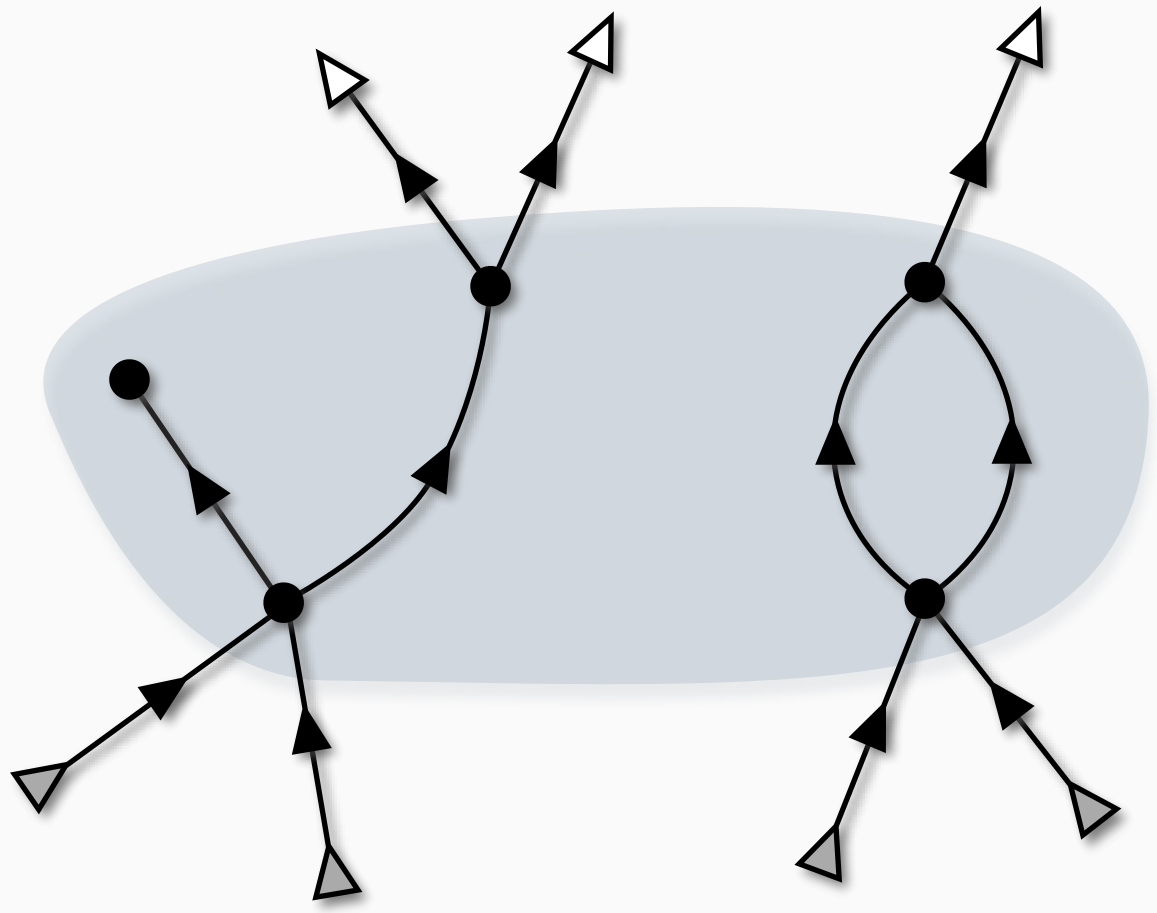
- *Edges in a graph may have one of the ends free (but not both)*
- *It has three sorts of edges:* ***inner****,* ***ingoing*** *and* ***outgoing*** *ones*
- *Size of a graph:* $d(\Gamma) = 2\,|\Gamma^0| + |\Gamma^+| + |\Gamma^-|$

Graph model

Vector space of graphs

We define $\mathcal{G}$ as a **vector space** over $\mathbb{C}$ spanned by the basis set consisting of all Heisenberg - Weyl graphs, i.e.

$$\mathcal{G} = \left\{ \sum_i \alpha_i \; \Gamma_i : \; \alpha_i \in \mathbb{C}, \; \Gamma_i \text{ - Heisenberg–Weyl graph} \right\}$$



Heisenberg - Weyl graph

Addition in $\mathcal{G}$ has the usual form:

$$\sum_i \alpha_i \; \Gamma_i + \sum_i \beta_i \; \Gamma_i = \sum_i (\alpha_i + \beta_i) \; \Gamma_i$$

What about the ***multiplication***?

$$\sum_i \alpha_i \; \Gamma_i \; * \sum_j \beta_j \; \Gamma_j = \sum_{i,j} \alpha_i \beta_j \; \Gamma_i * \Gamma_j$$

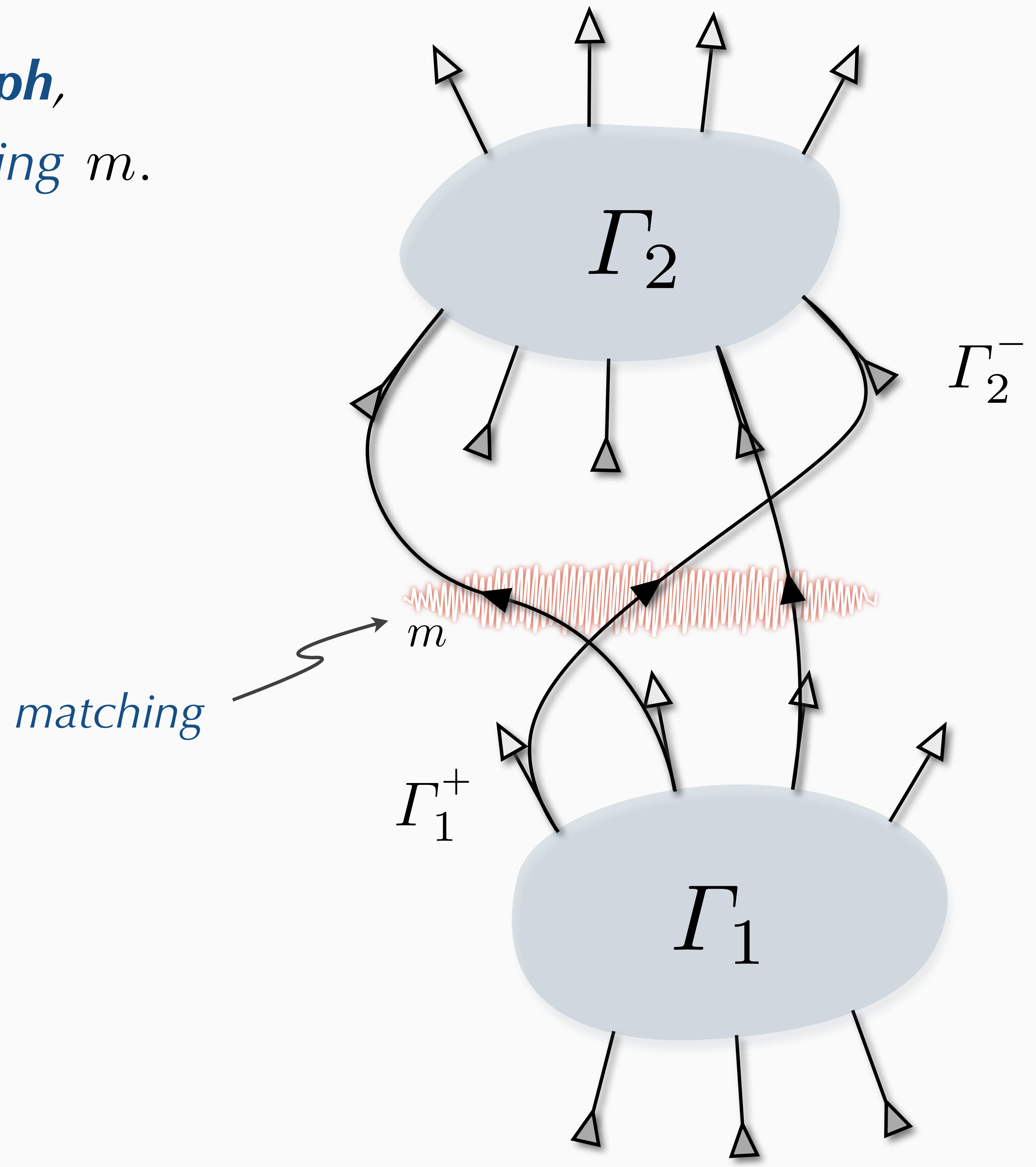
Definition:

For two graphs Γ_2 and Γ_1 and a **matching** $m \in \Gamma_2^- \lll \Gamma_1^+$ the **composite graph**, denoted as $\Gamma_2 \overset{m}{\blacktriangleleft} \Gamma_1$, is constructed by joining the edges coupled by the matching m.

Definition:

Multiplication of two graphs Γ_2 and Γ_1 in $\mathcal{G}$ is just a sum over all possible compositions:

$$\Gamma_2 * \Gamma_1 = \sum_{m \in \Gamma_2^- \lll \Gamma_1^+} \Gamma_2 \overset{m}{\blacktriangleleft} \Gamma_1$$



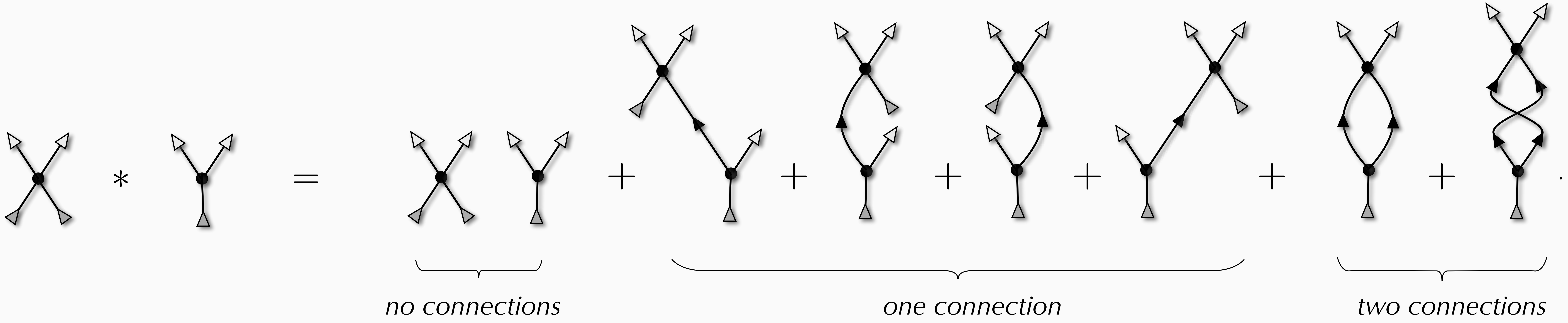
Proposition:

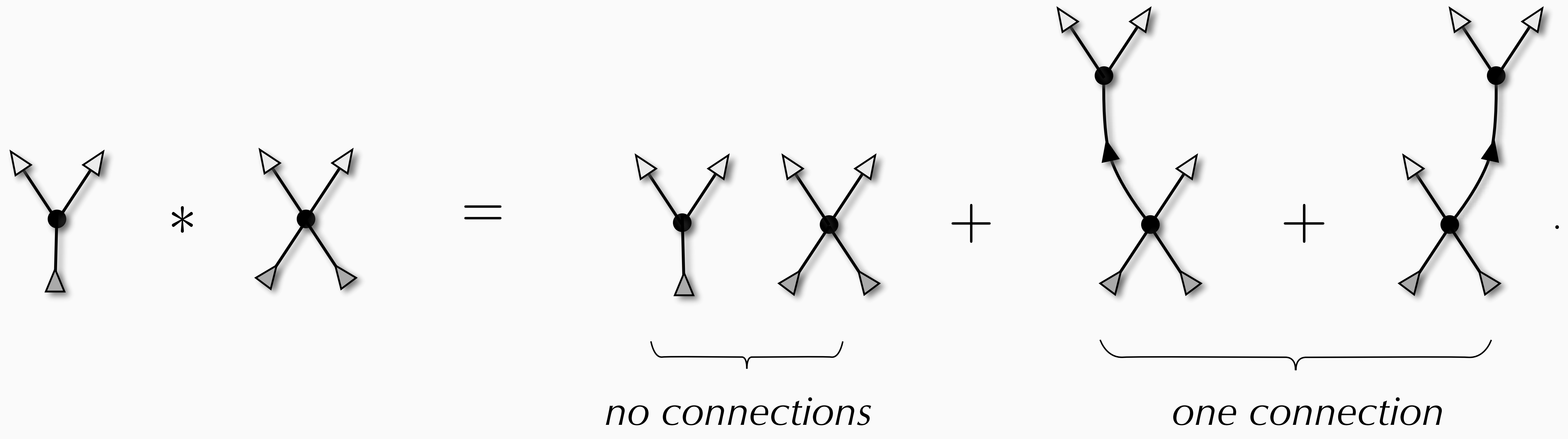
Heisenberg - Weyl graphs form an **associative algebra with unit** $(\mathcal{G}, +, *, \emptyset)$.
It is non-commutative !!

- The number of possible compositions with i connections: $\# \, \Gamma_2^- \overset{i}{\lll} \Gamma_1^+ = \binom{|\Gamma_2^-|}{i}\binom{|\Gamma_1^+|}{i} \, i!$

Graph model

Example





Graph model

Model of the Heisenberg-Weyl algebra

Definition:

We define a linear mapping $\varphi : \mathcal{G} \longrightarrow \mathfrak{H}$ *which* ***erases all inner structure*** *of a graph, given on the basis elements as:*

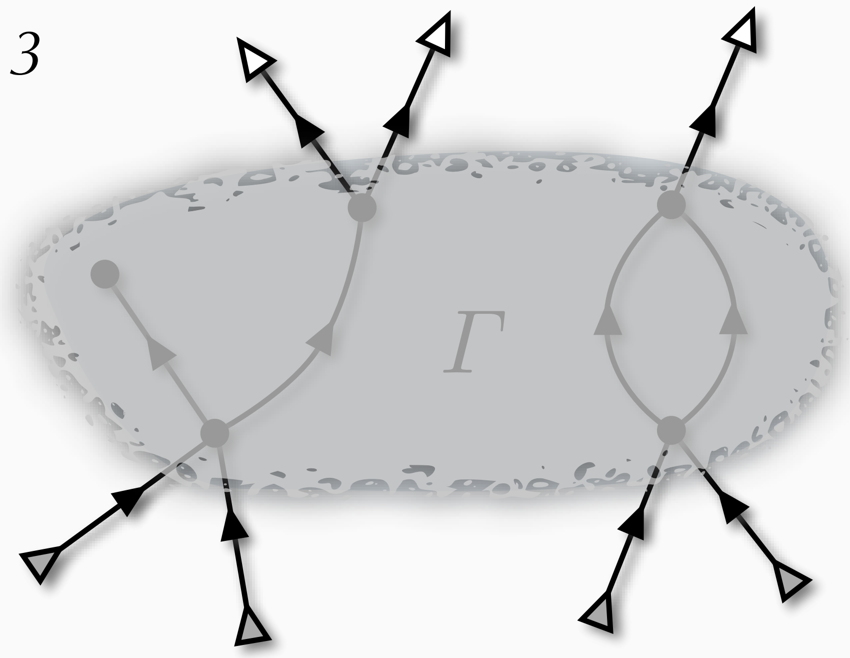
$$\varphi(\Gamma) = X^{|\Gamma^{+}|} D^{|\Gamma^{-}|}$$

Example:

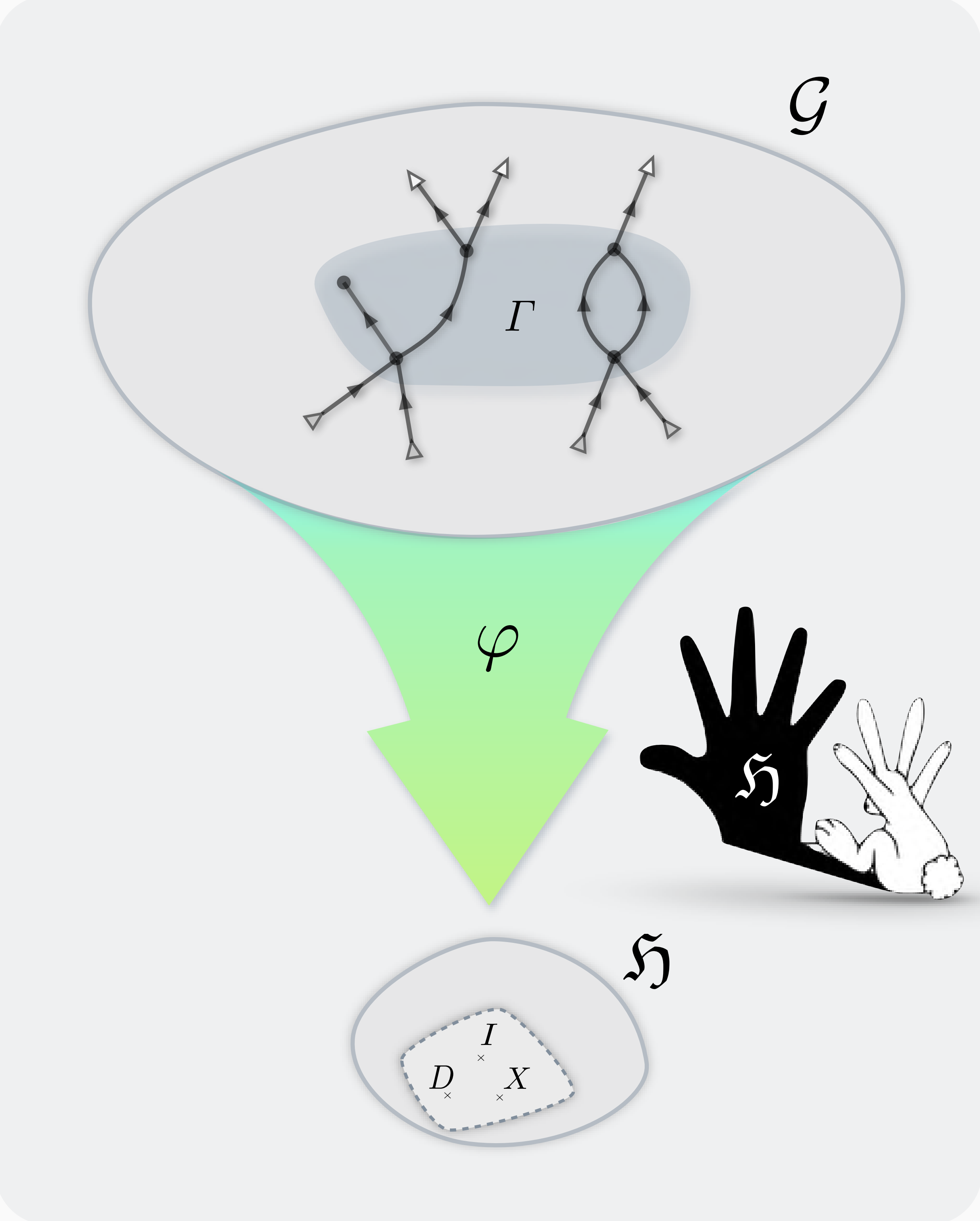
Outgoing: 3

Inner: 4

Ingoing: 4



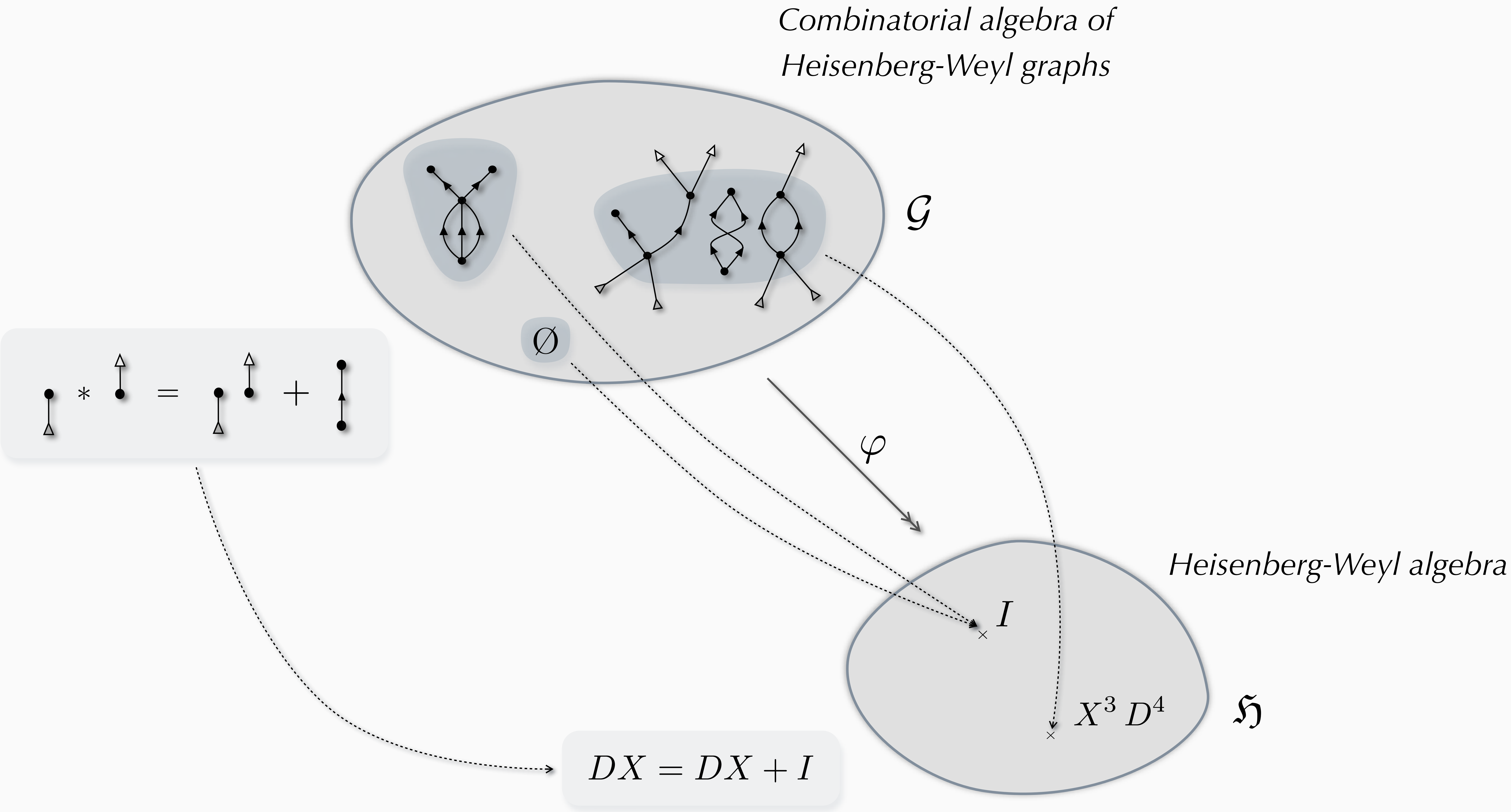
$$\xrightarrow{\ \varphi\ } \quad X^3 D^4$$

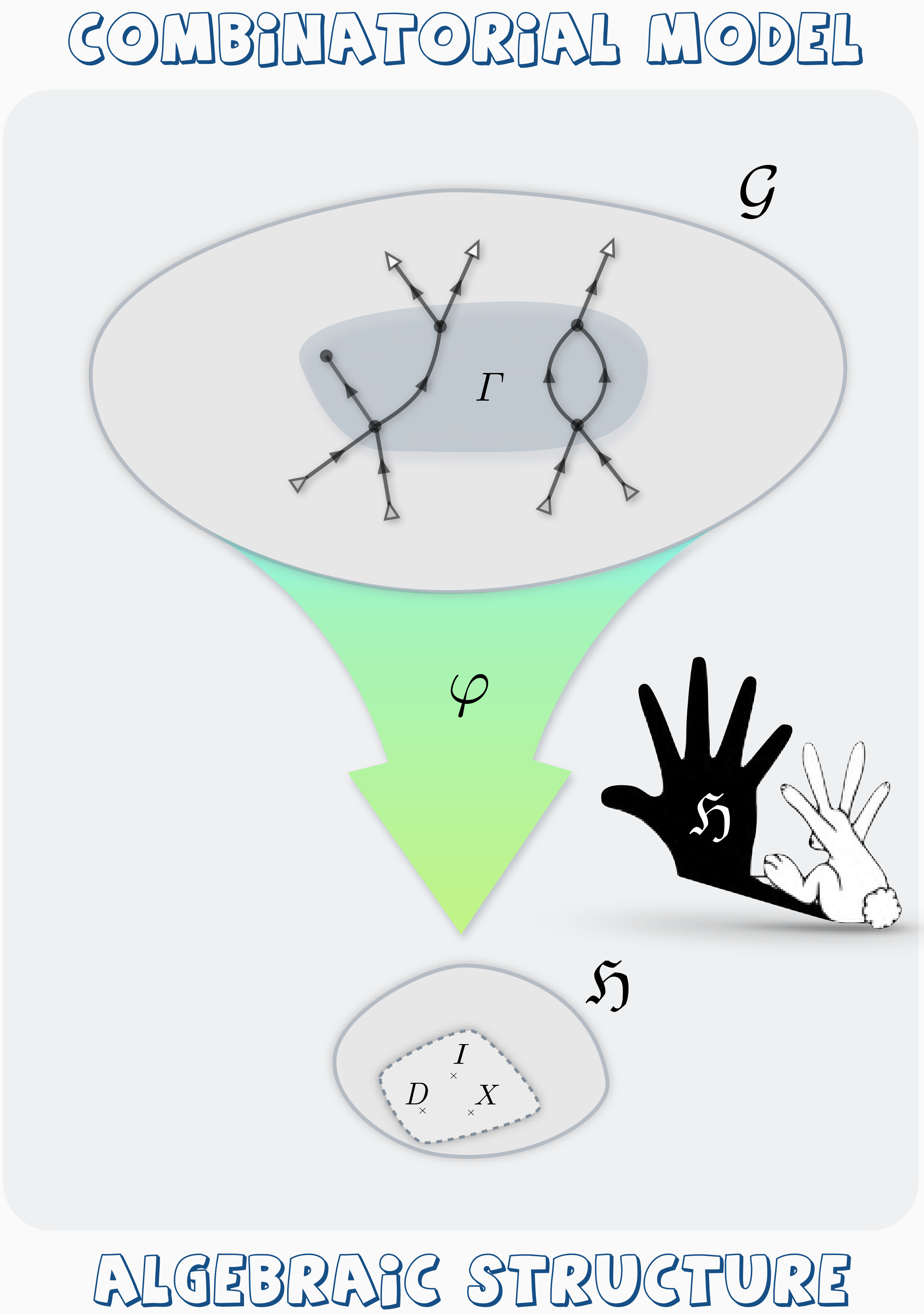


Theorem:

Forgetful mapping $\varphi : \mathcal{G} \longrightarrow \mathfrak{H}$ *is a surjective AAU* ***algebra morphism***.

Graph model
Example



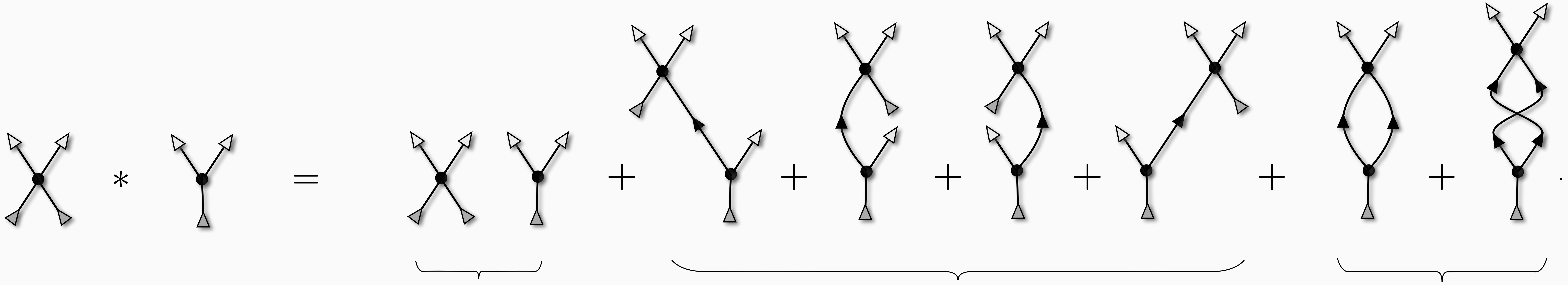


Morphism $\varphi : \mathcal{G} \longrightarrow \mathfrak{H}$ **erases all inner structure** of a graph, and **preserves all relations**.

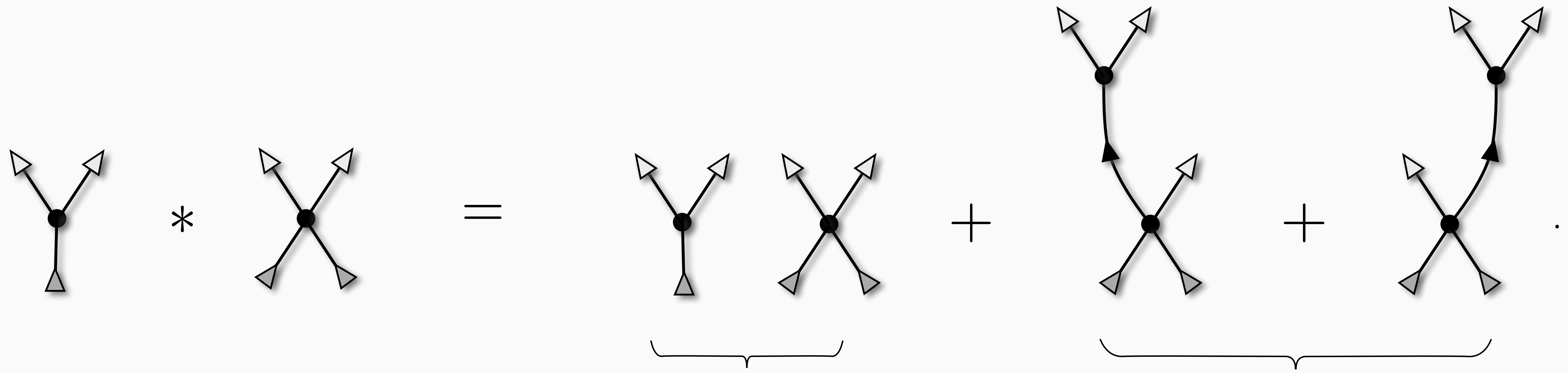
P. Blasiak, G. H. E. Duchamp, A. I. Solomon, A. Horzela, and K. A. Penson. **Combinatorial Algebra for second-quantized Quantum Theory**. Adv. Theor. Math. Phys., 14(4):1209–1243, 2010

Graph model

Example



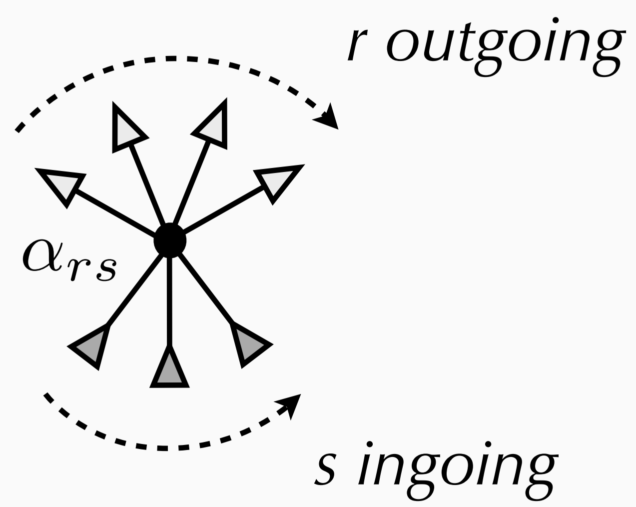
$$X^2 D^2\, X^2 D = X^4 D^3 + 4\, X^3 D^2 + 2\, X^2 D$$

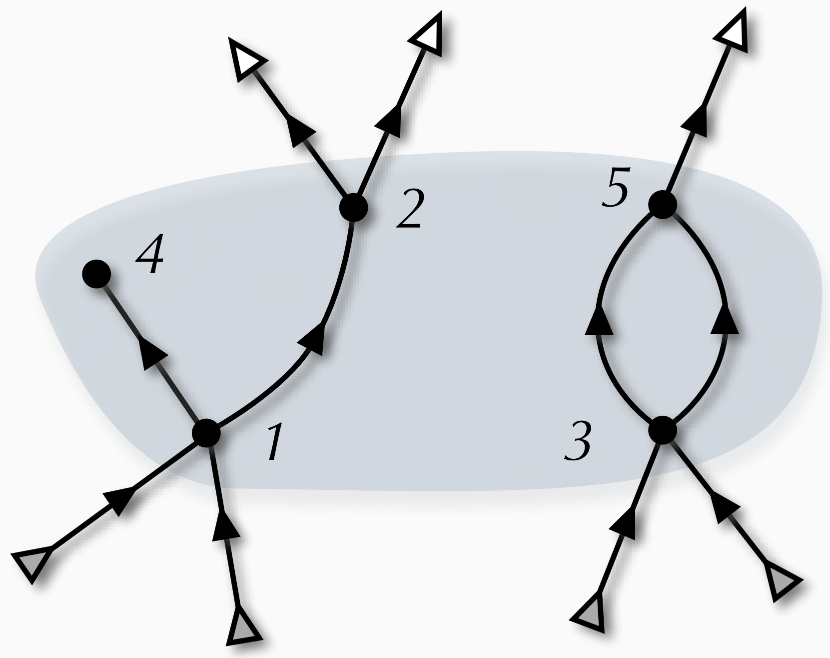


$$X^2 D\, X^2 D^2 = X^4 D^3 + 2\, X^3 D^2$$

Building a graph
Gates and graph labeling

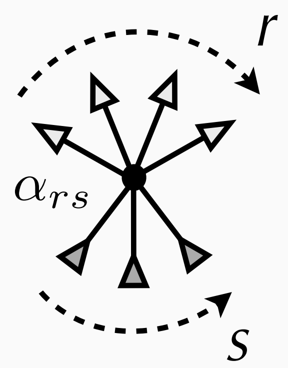
*Building blocks of a graph (**gates**):*





- ● - atoms z
- △ - marker u
- △ - marker v

*Each choice of gates ("**basis**" **set**):*



$$\mathfrak{h} \subset \left\{ \quad : r, s \in \mathbb{N} \;,\; r, s \neq 0 \right\}$$

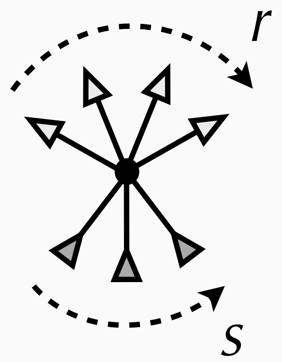
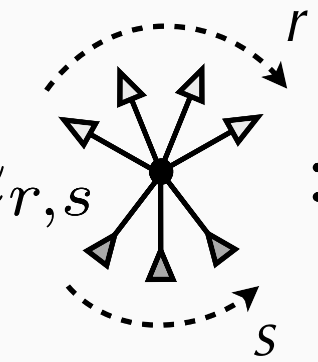
generates a combinatorial class of graphs which have all vertices of type $\mathfrak{h}$.

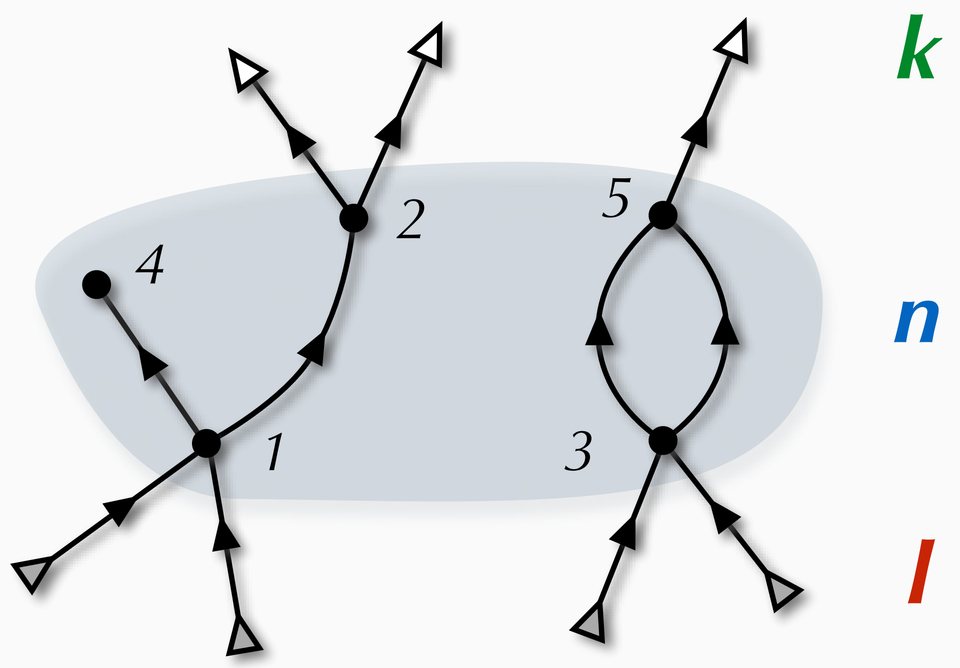
Additional structure:

- *Can attach **multiplicative weights** α_{rs} to vertices.*
- ***Increasing labelling** is an additional structure on graphs introduced by labelling vertices with 1,2,3,… such that labels are increasing along each directed path.*

Enumeration of Graphs

and normal ordering

For $\Gamma = \sum_{r,s} \alpha_{r,s}$  consider the associated "**basis**" **set**: $\mathfrak{h} = \{\alpha_{r,s}$  $: \alpha_{r,s} \neq 0\}$



- *On the level of graphs $\mathcal{G}$:* $\quad \Gamma^{n} = \sum$ *increasingly labelled graphs built of **n** vertices of types* $\mathfrak{h}$

- *On the level of algebra $\mathfrak{H}$:*

$$\left(\sum_{r,s} \alpha_{r,s}\, X^r D^s\right)^{n} = \sum_{k,l} \overbrace{\#\left[\begin{array}{c}\text{increasingly labelled graphs}\\ \text{built of } n \text{ vertices of types } \mathfrak{h}\\ \text{with } k \text{ outgoing and } l \text{ ingoing lines}\end{array}\right]}^{\beta_{n,k,l}}\; X^{k} D^{l}$$

$$\exp\left(z \sum_{r,s} \alpha_{r,s}\, X^r D^s\right) = \sum_{n,k,l} \beta_{n,k,l}\, u^{k} v^{l} \frac{z^{n}}{n!} = G(z;u,v)\Big|_{\substack{u \to X\\ v \to D}}$$

Normal form !!!

- ● - *atoms* z
- △ - *marker* u
- ▲ - *marker* v

Equivalence principle

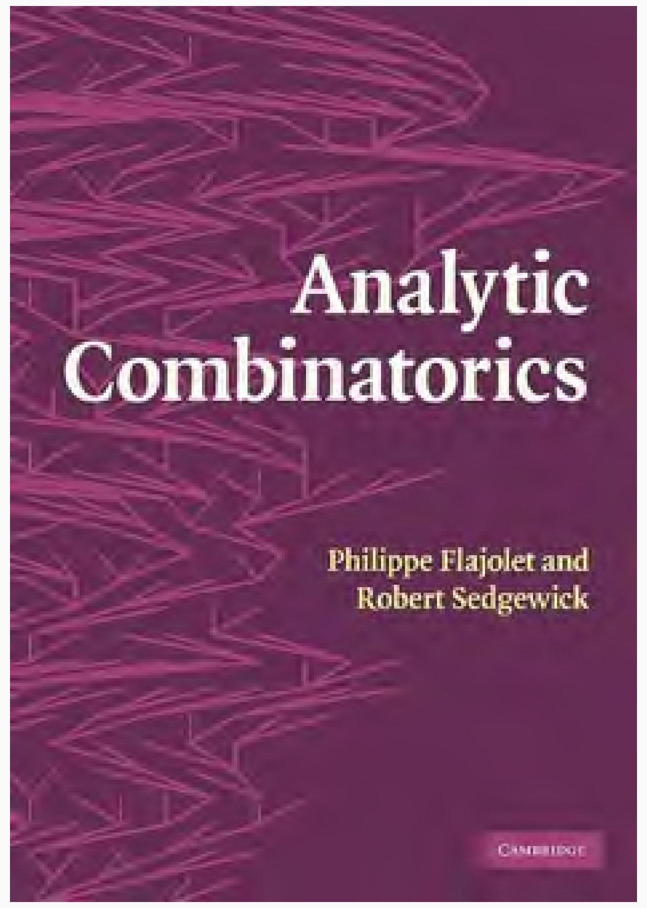
Symbolic Methods & Generating Functions

Constructive approach to normal ordering

$\mathcal{C}$ - combinatorial class (collection of objects with the notion of size)

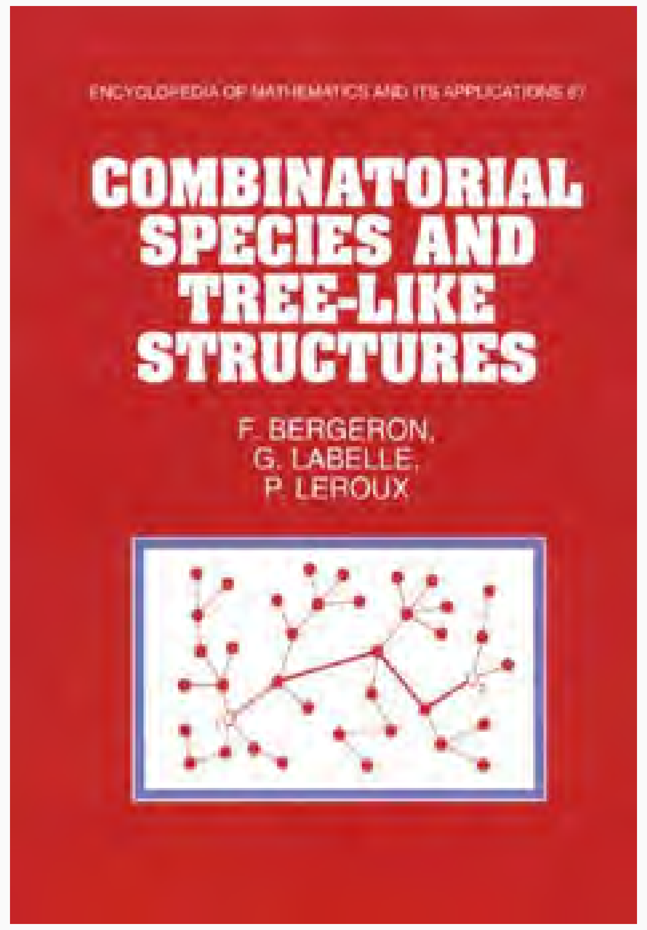
$C_n = \#$ objects of size n in class $\mathcal{C}$

$C(z) = \sum_n C_n \frac{z^n}{n!}$ - exponential generating function of class $\mathcal{C}$



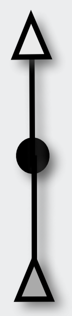
- Set-theoretic constructions translate into generating functions.

Disjoint union:	$\mathcal{C} = \mathcal{A} \uplus \mathcal{B}$	$\Longrightarrow$	$C(z) = A(z) + B(z)$
Cartesian product:	$\mathcal{C} = \mathcal{A} \times \mathcal{B}$	$\Longrightarrow$	$C(z) = A(z) \cdot B(z)$
Set construction:	$\mathcal{C} = \textsc{Set}\,(\mathcal{A})$	$\Longrightarrow$	$C(z) = e^{A(z)}$
Substitution:	$\mathcal{C} = \mathcal{A} \circ \mathcal{B}$	$\Longrightarrow$	$C(z) = A\big(B(z)\big)$
Appending min/max element:	$\mathcal{C} = \mathcal{Z}^{\Box} \star \mathcal{A}$	$\Longrightarrow$	$C(z) = \int_0^z A(t)\, dt$
..... and many others, e.g.	$\textsc{Seq}\,,\ \textsc{Cyc}\,,\ \mathrm{D}\,,\ \Theta\,,\ \ldots$		

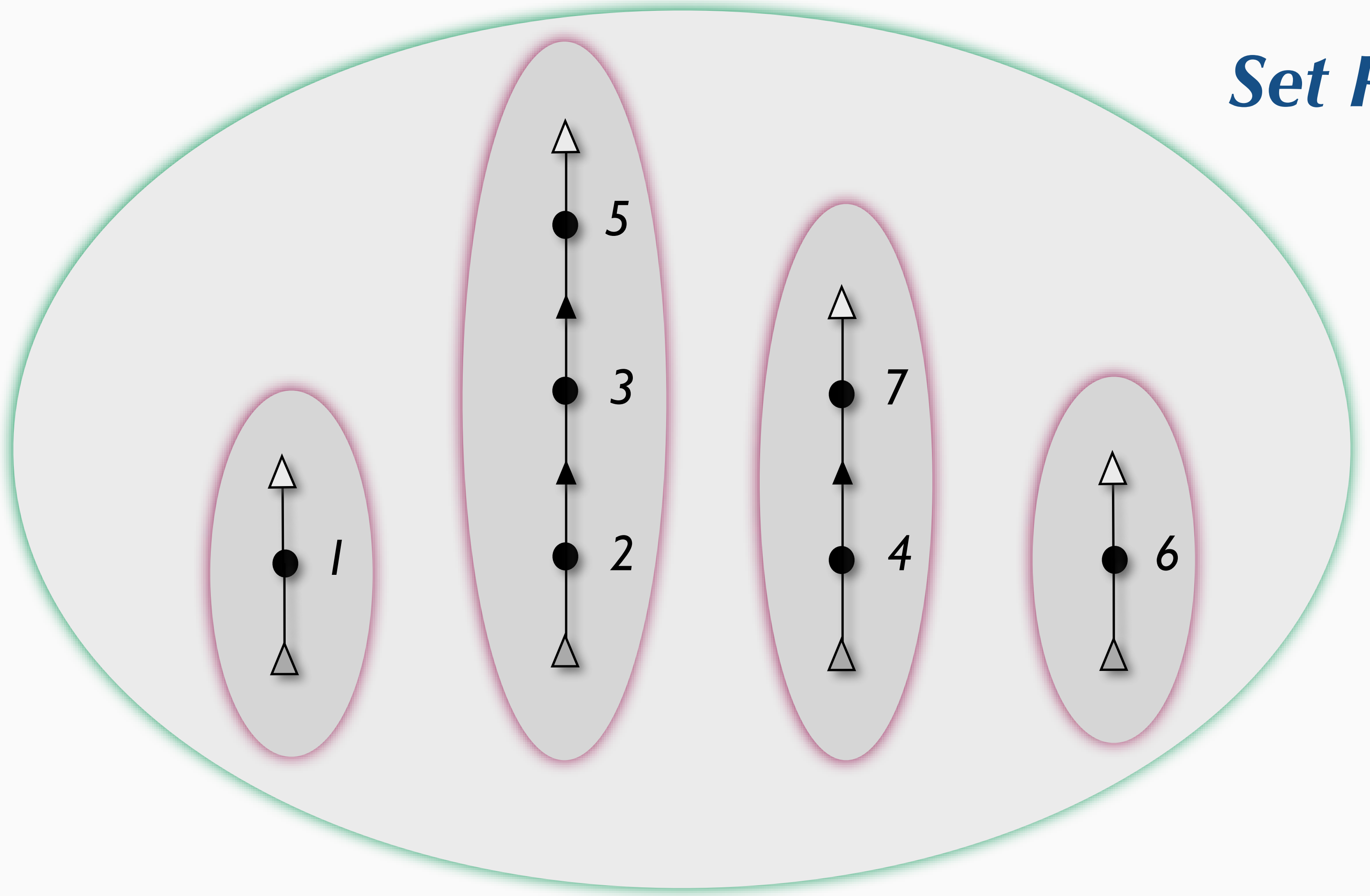


Combinatorial structures

Example: Set partitions

Let us take  , i.e. XD.

Generic graph:



Set Partitions

● - atoms z

△ - marker u

▲ - marker v

Combinatorial specification: $\mathcal{C} = \textsc{Set}\left(uv\,\textsc{Set}_{\geqslant 1}(\mathcal{Z})\right)$

Generating function: $G = e^{uv(e^z-1)}$

On the algebraic level: $\mathfrak{N}\left(e^{z\,XD}\right) = \,:e^{XD\,(e^z-1)}:$

Combinatorial structures

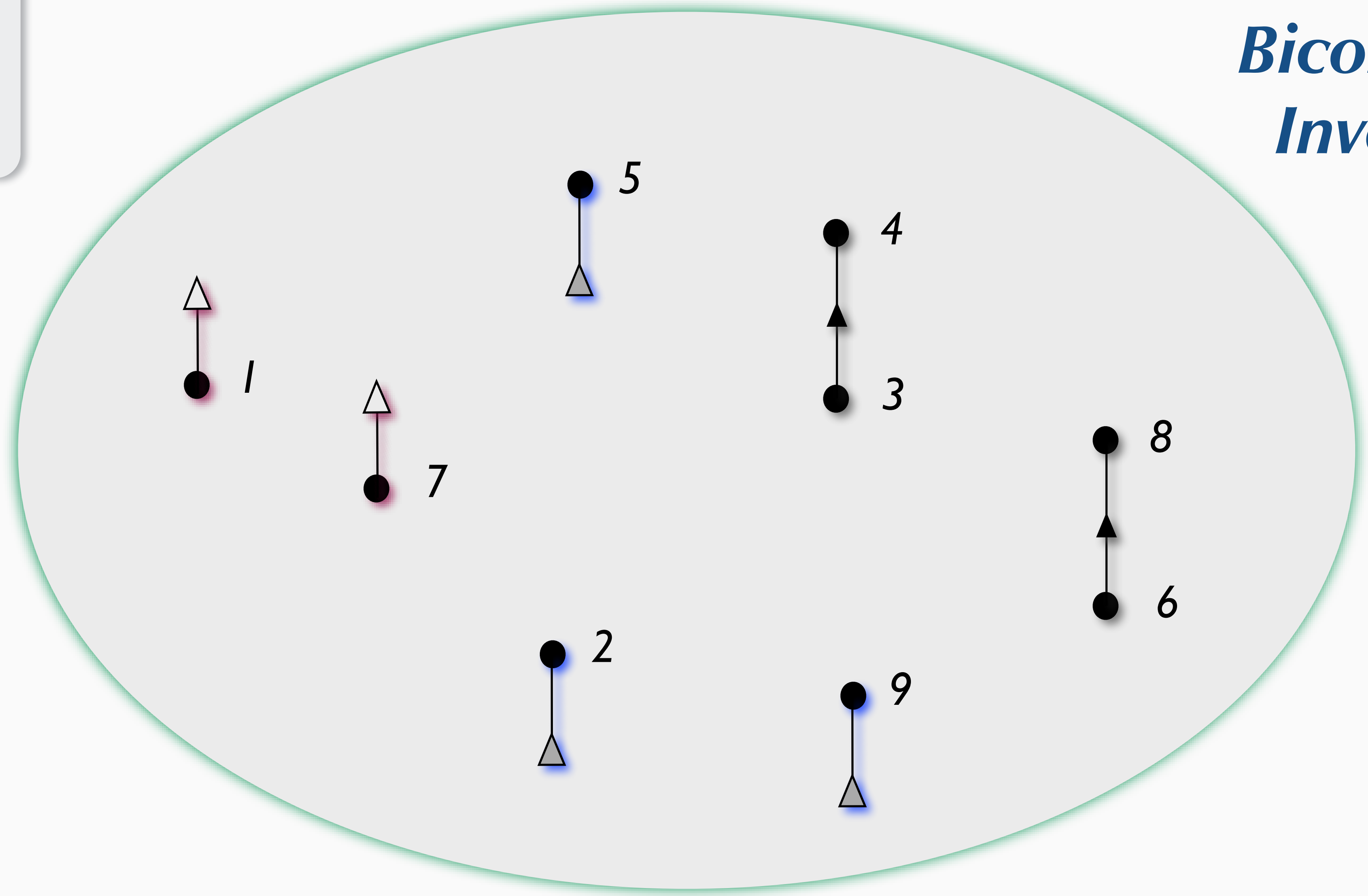
Example: Involutions

Let us take  $+$  *, i.e.* $X + D$.

- ● - *atoms* z
- △ - *marker* u
- △ - *marker* v

Bicoloured Involutions

Generic graph:



Combinatorial specification: $\mathcal{C} = \textsc{Set}\,(\,u\,\mathcal{Z} + v\,\mathcal{Z} + \textsc{Set}_2\,(\mathcal{Z}))$

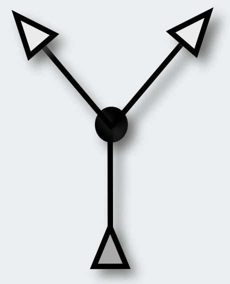
Generating function: $G = e^{(u+v)z + z^2/2}$

On the algebraic level: $\mathfrak{N}\big(e^{z\,(X+D)}\big) = \,:e^{(X+D)z + z^2/2}:\, = \; e^{z^2/2} \cdot e^{z\,X} \cdot e^{z\,D}$

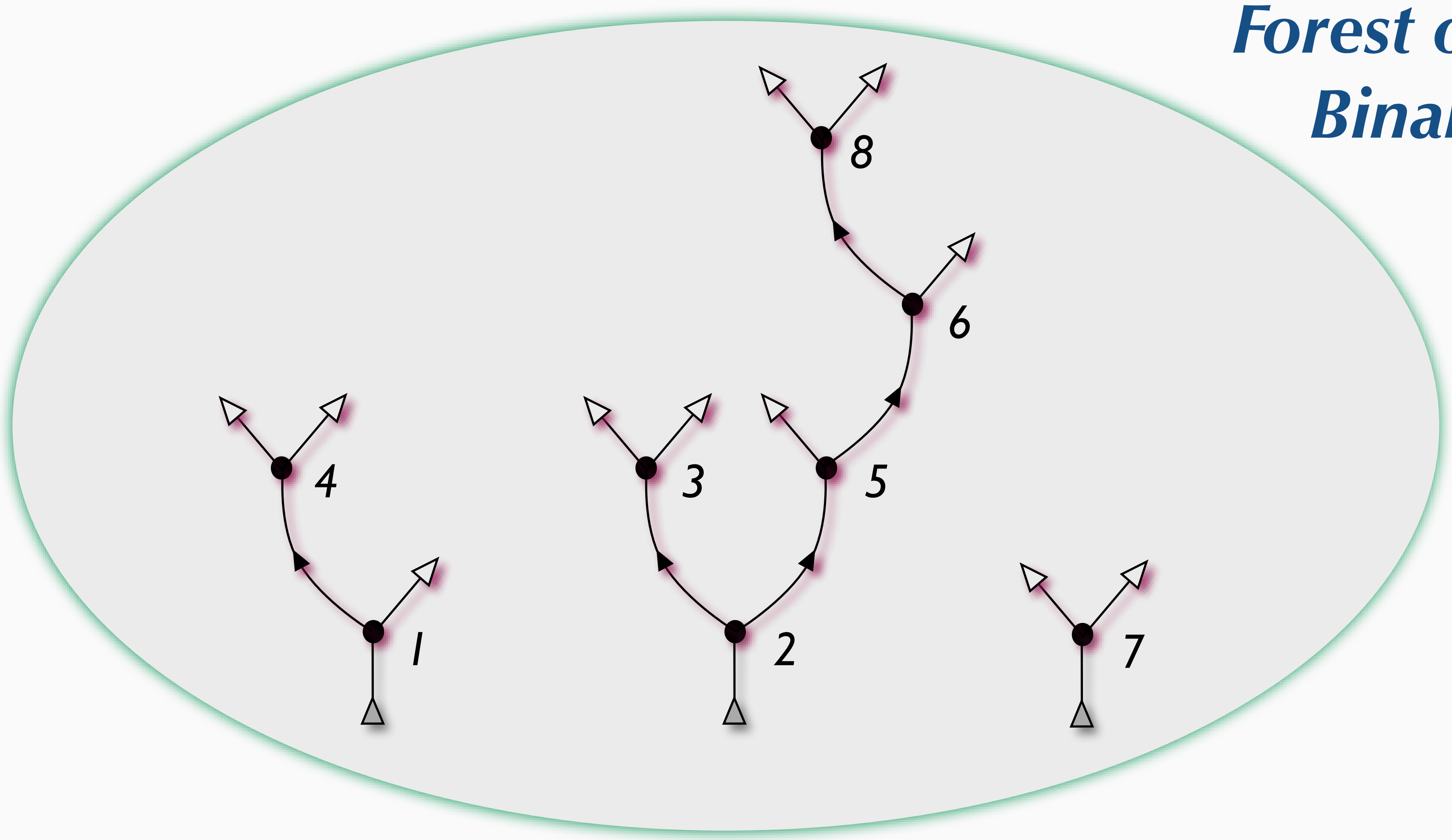
!! BCH formula !!

Combinatorial structures

Example: Binary trees

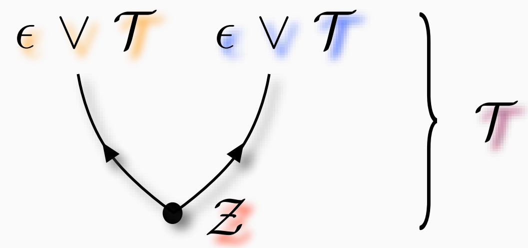
Let us take  , i.e. $X^2 D$.

Generic graph:



- $\bullet$ - atoms z
- $\triangle$ - marker u
- $\triangle$ - marker v

Construction of a Tree:



Combinatorial specification:

$$\mathcal{C} = \textsc{Set}\,(\,v\,\mathcal{T}\,)$$

$$\begin{cases} \mathcal{T} = (u + \mathcal{T}) \star \mathcal{Z}^{\Box} \star (u + \mathcal{T}) \\ \mathcal{T}(0) = 0 \end{cases}$$

Generating function:

$$G = \exp\left(\frac{v\,u^2\,z}{1 - u\,z}\right)$$

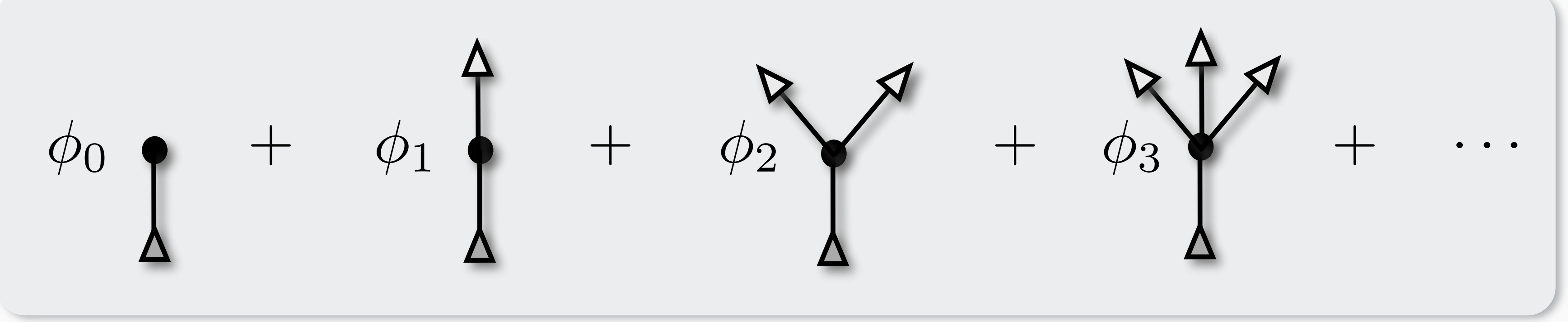
$$T(u,z) = \int_0^z (u + T(u,t))^2 \; dt$$

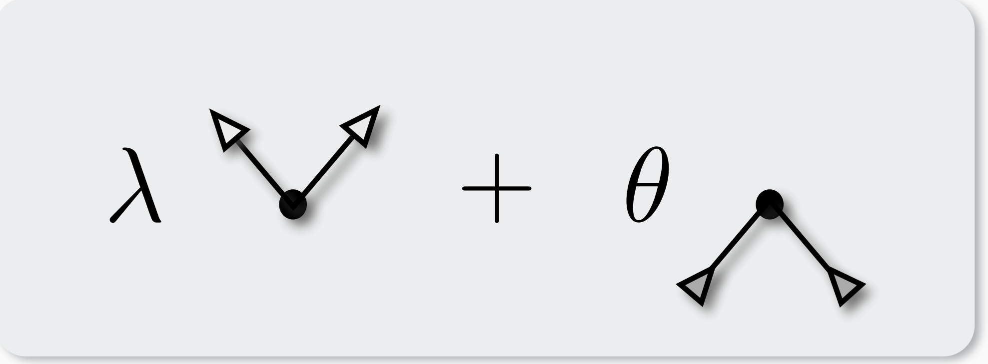
On the algebraic level:

$$\mathfrak{N}(e^{\,z\,X^2 D}) = :\exp\left(\frac{X^2 D\,z}{1 - X\,z}\right):$$

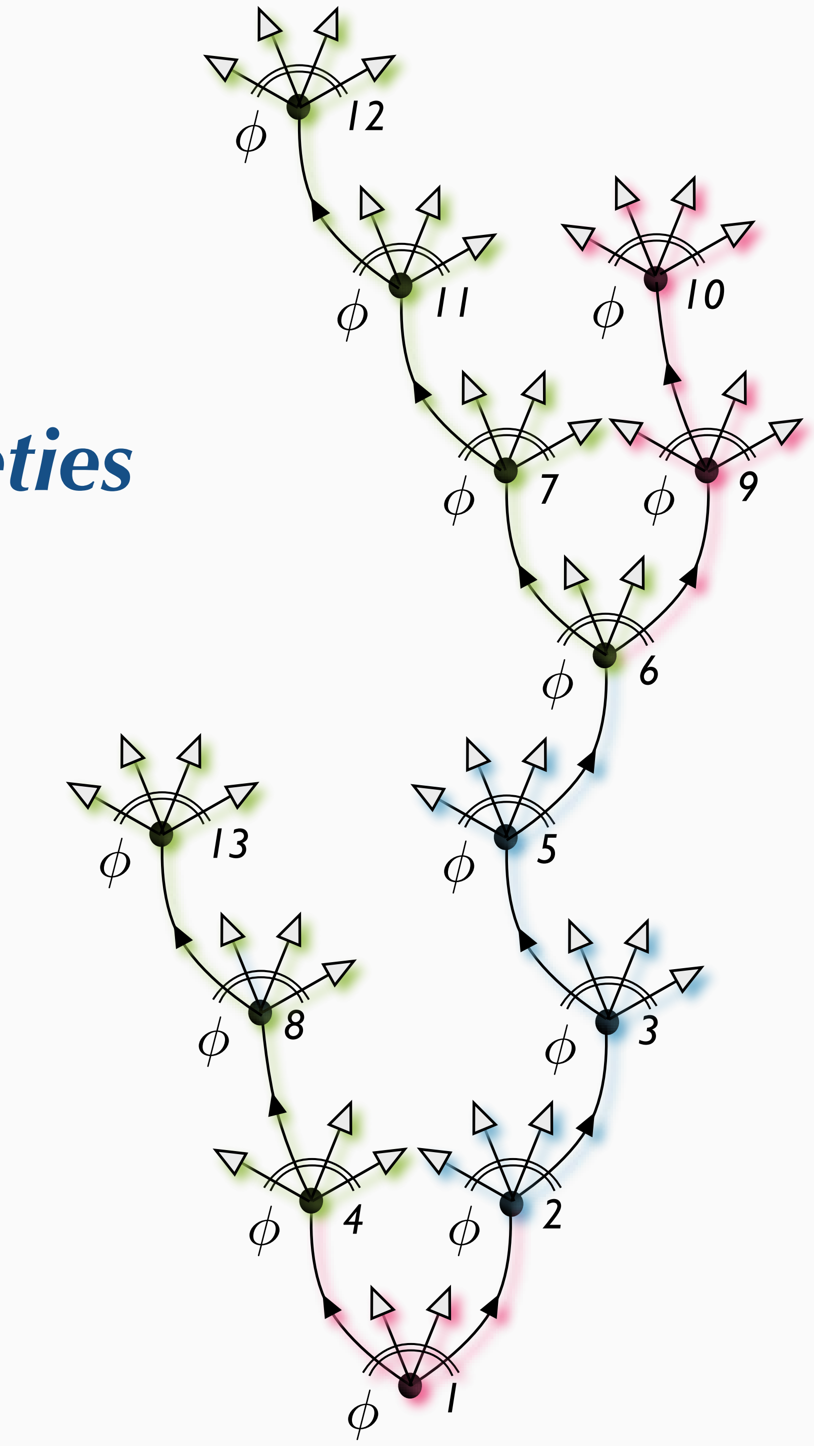
Combinatorial structures

and much more …





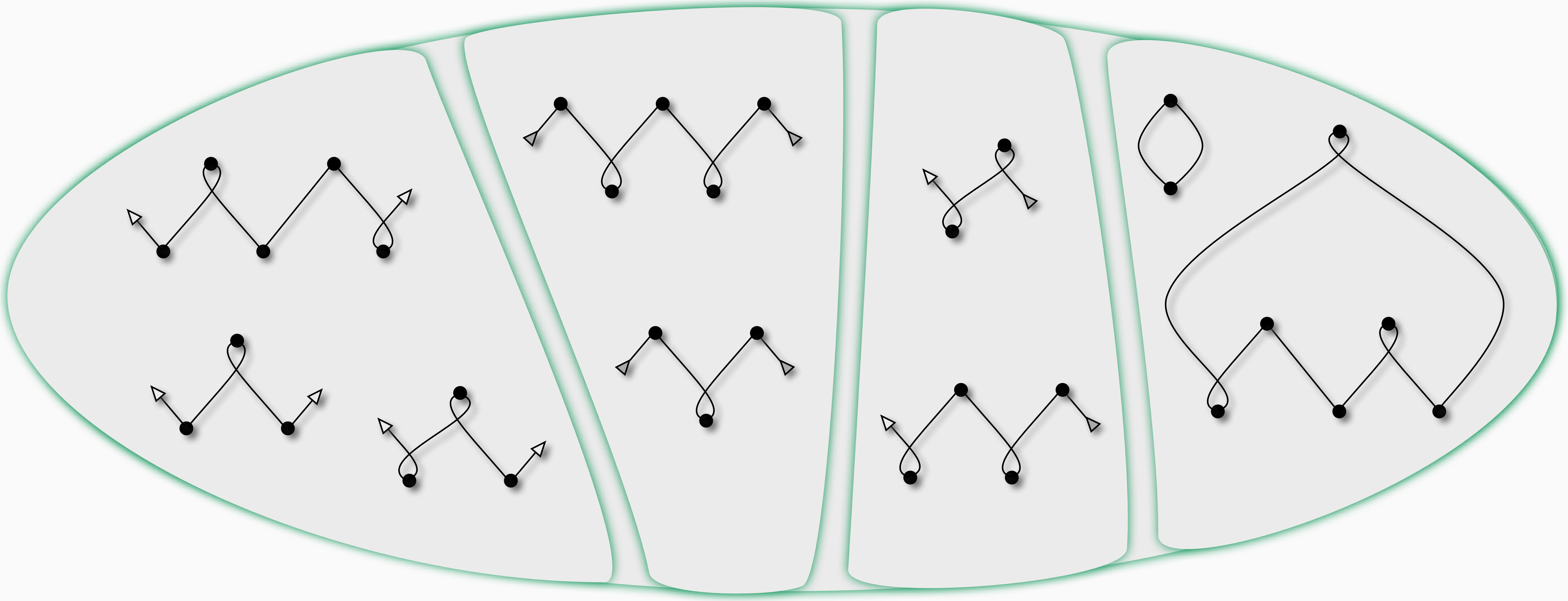
Tree varieties



$$\mathfrak{N}\left(e^{z\,\phi(X)D}\right) = e^{v\,T^{\phi}(u,z)}$$

Open/closed ZIG-ZAGS

(alternating) permutations



$$\mathfrak{N}\left(e^{\lambda X^2+\theta D^2}\right) = \sqrt{\sec(\sqrt{4\lambda\theta})}\;\; e^{\sqrt{\frac{\lambda}{4\theta}}\,\tan(\sqrt{4\lambda\theta})\,X^2} : e^{\left(\sec(\sqrt{4\lambda\theta})-1\right)\,XD} : \; e^{\sqrt{\frac{\theta}{4\lambda}}\,\tan(\sqrt{4\lambda\theta})\,D^2}$$

Back to Quantum Foundations

A naive attempt at interpretation

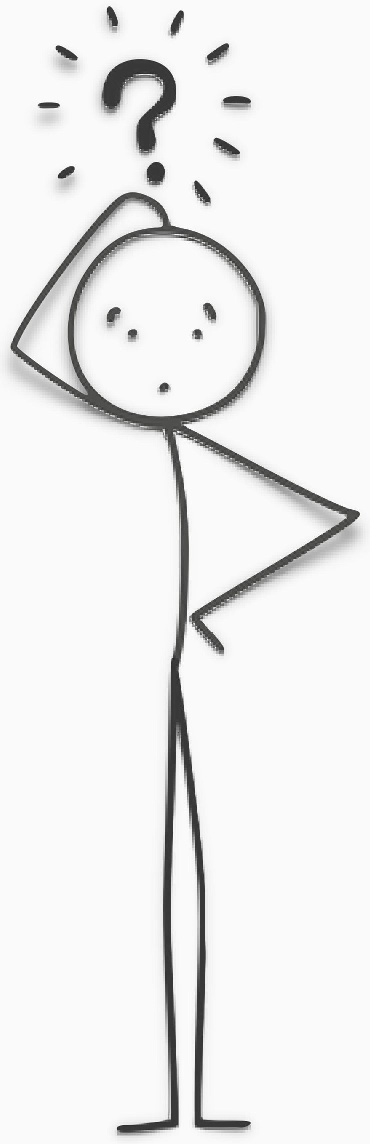
Some insight into quantum evolution (Schrödinger equation)

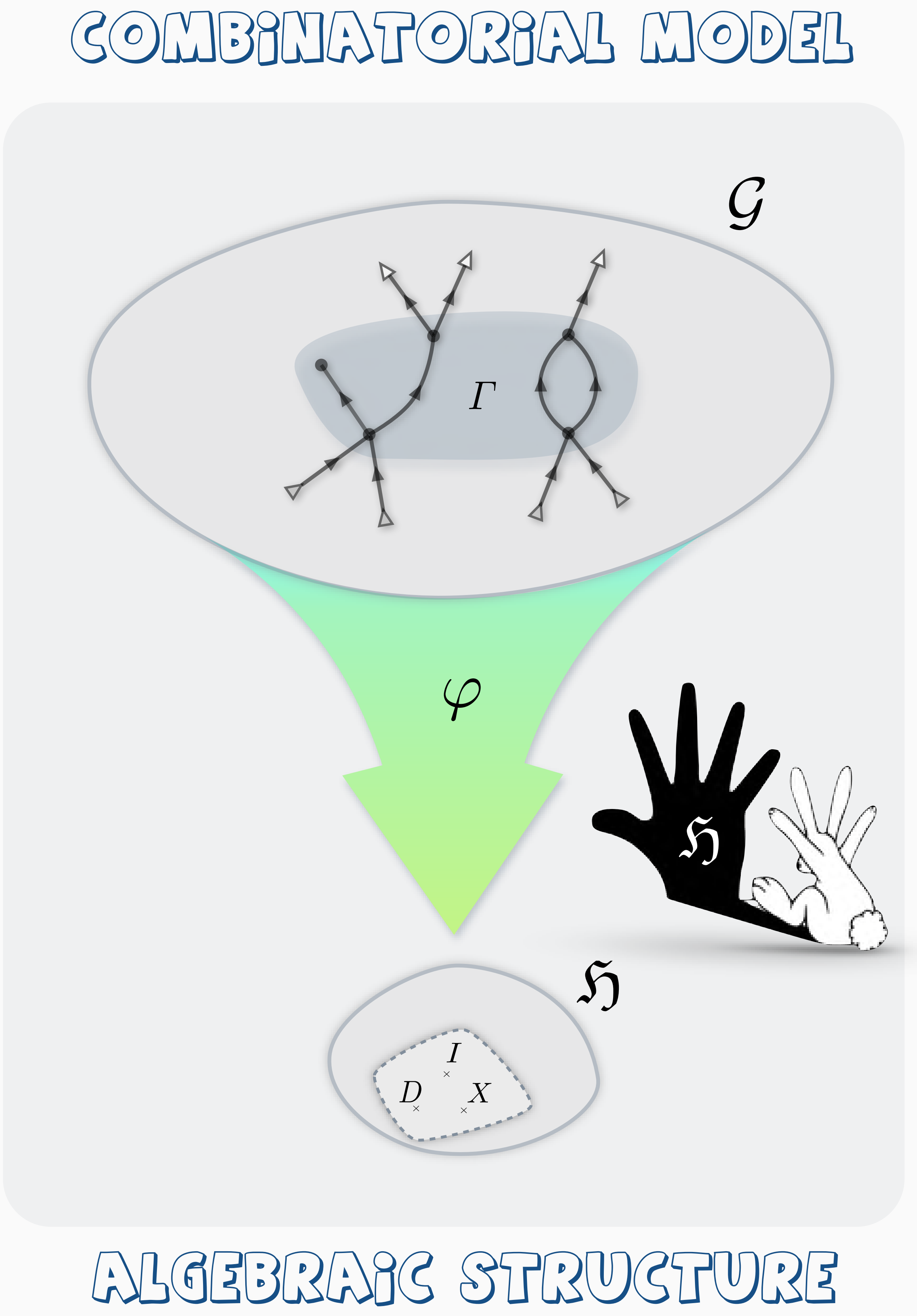
$$|\Psi_0\rangle \longrightarrow |\Psi_t\rangle = e^{itH(a,a^\dagger)}\,|\Psi_0\rangle \qquad \text{where:} \quad |\Psi\rangle = \sum_n \alpha_n\,|n\rangle$$

$$\Psi_0(x) \longrightarrow \Psi_t(x) = e^{itH(\partial_x,x)}\,\Psi_0(x) \qquad \text{where:} \quad \Psi(x) = \sum_n \alpha_n\,x^n$$

$$\mathcal{C}_0 \longrightarrow \mathcal{C}_t = e^{itH(D,X)}\,\mathcal{C}_0 \qquad \text{where:} \quad \mathcal{C} \text{ - combinatorial class}$$

Can think as ***constructors acting on generating functions of combinatorial classes****?*



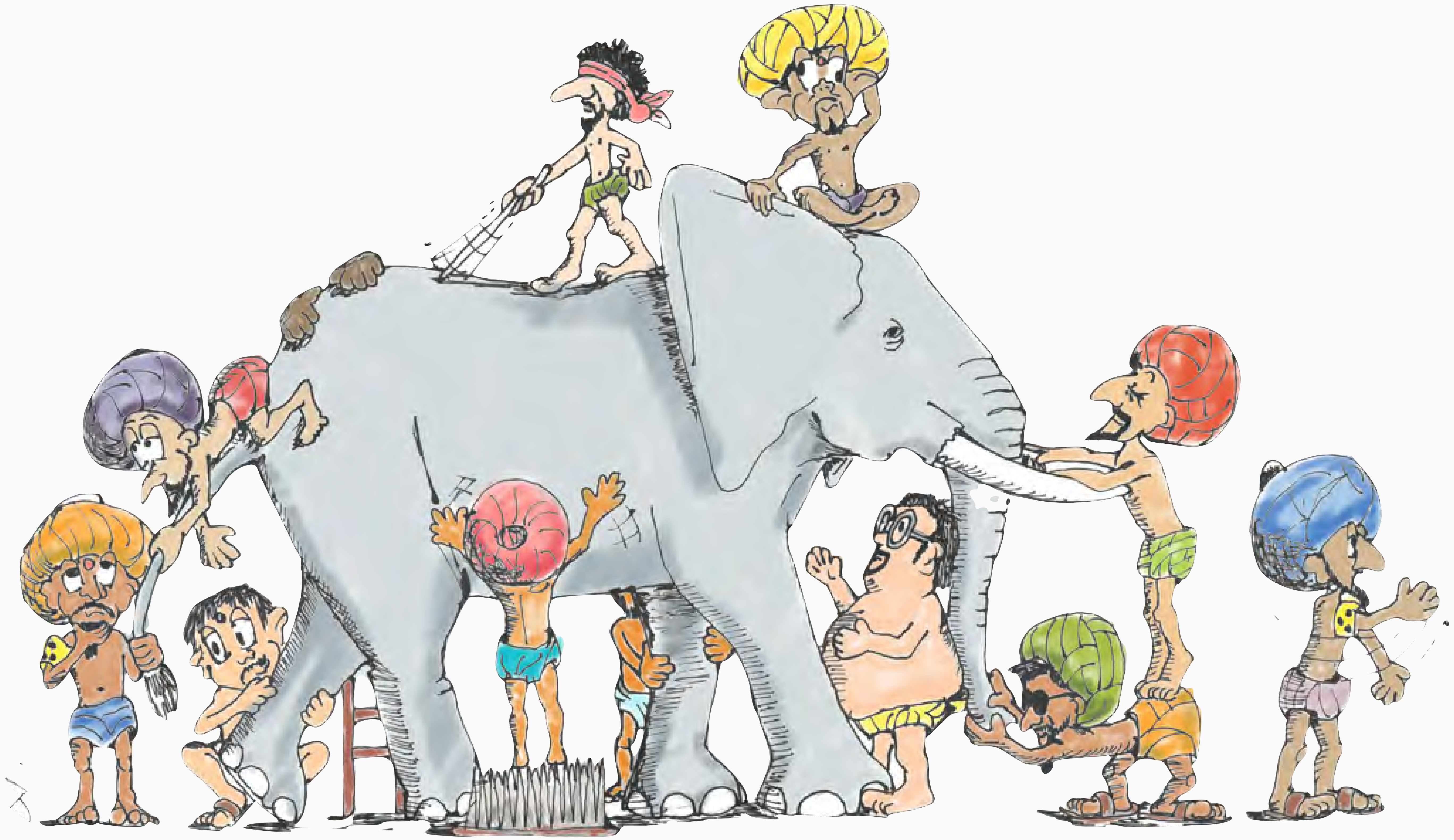


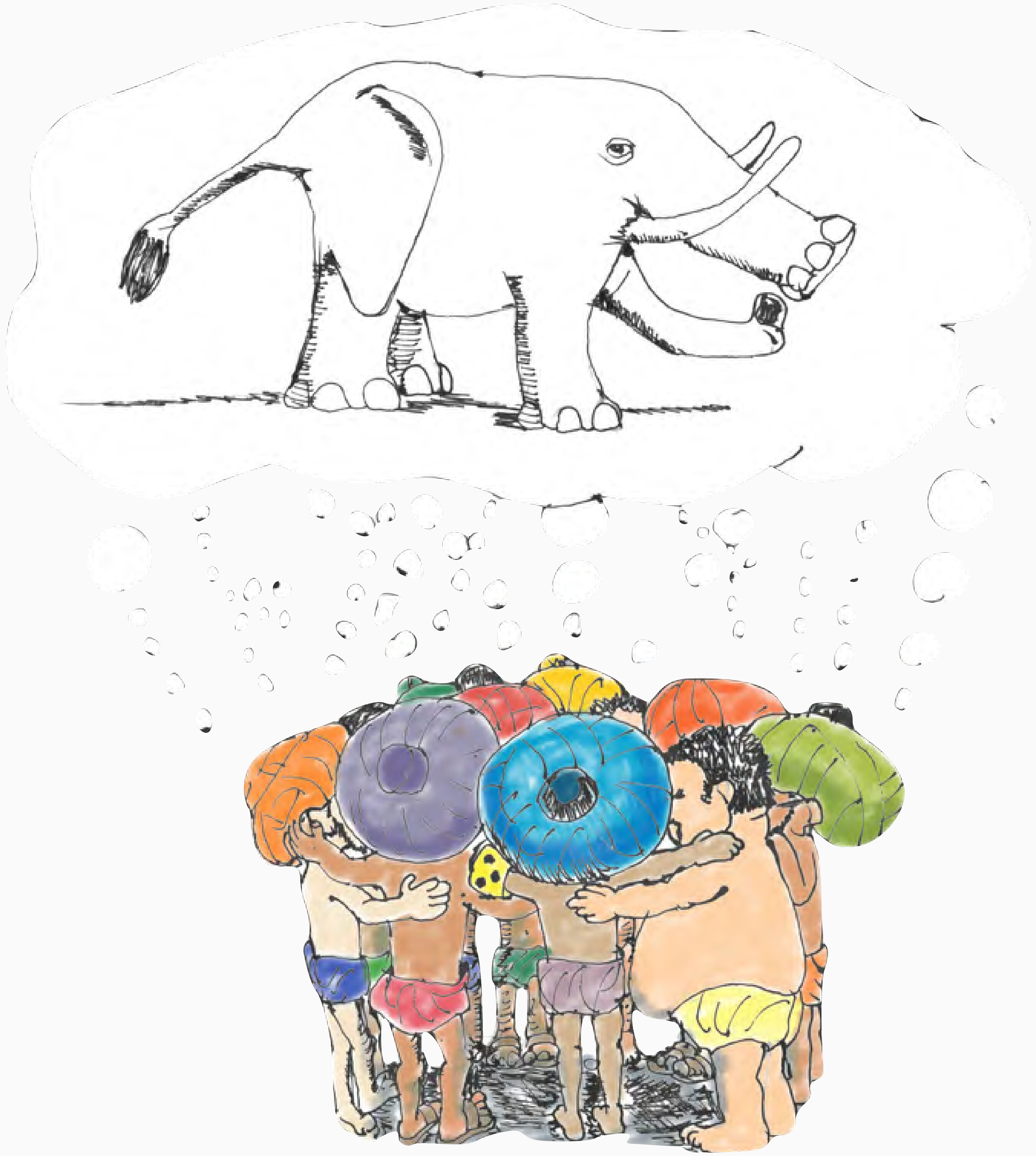
Problematic:

- *What is the meaning of g.f.* ***evaluated*** *at a point?*
- *How to derive/understand the* ***Born's rule****?*
- *What is the meaning of* ***complex weights*** *(interference phenomena)?*
- *Action on* ***whole classes*** *(no interpretation in terms of action on single objects)*

Blind man and an elephant

Information is physical





*"We have to remember that what we observe is not nature in itself, but **nature exposed to our method of questioning**."*

— Werner Heisenberg

Quantum formalism

Qubit and the Bloch ball representation

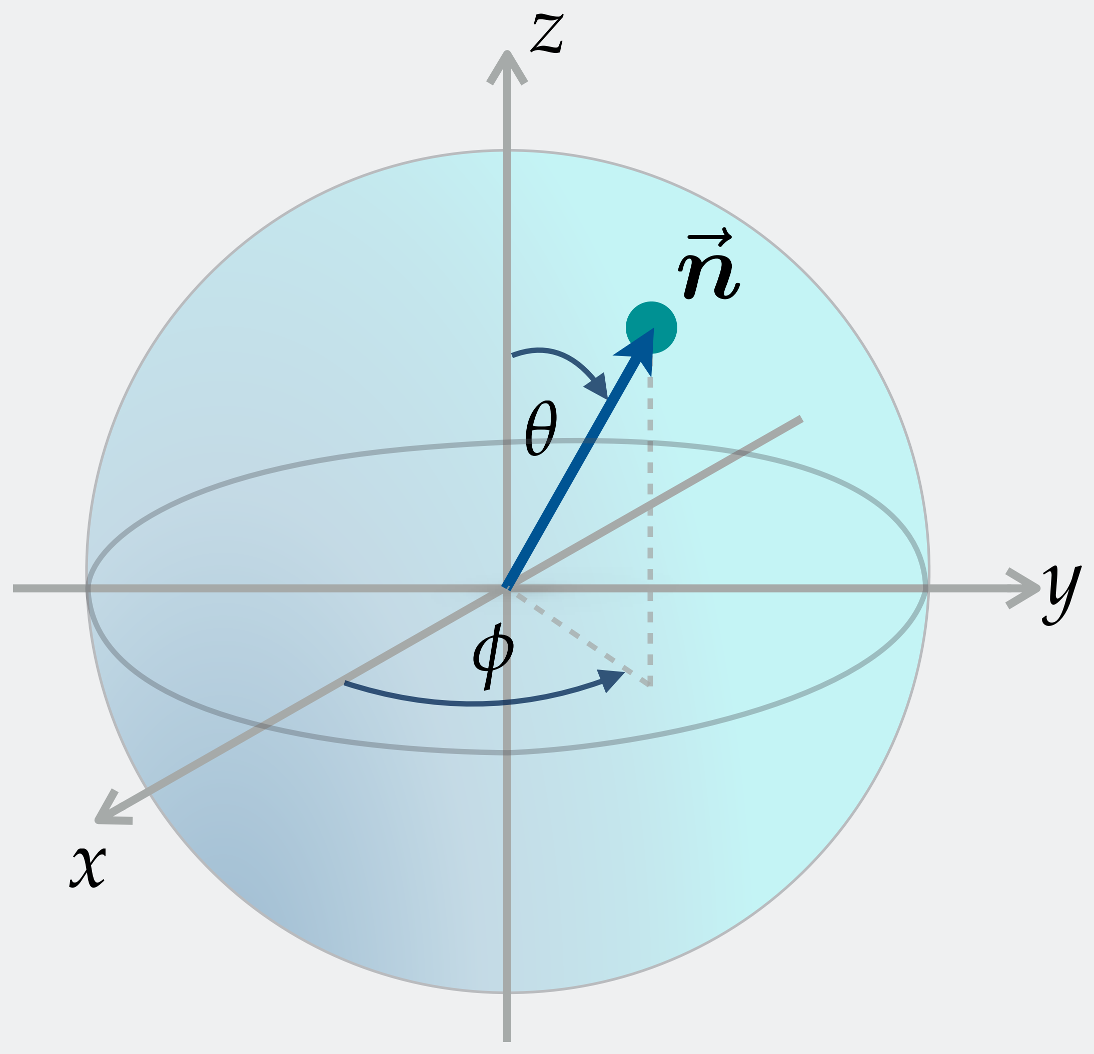
- *Representation of a* ***qubit****:* $\mathcal{H} = \mathbb{C}^2$

  - ***Pure*** *states:*

$$|\psi\rangle = \cos\left(\tfrac{\theta}{2}\right)|0\rangle + e^{i\phi}\sin\left(\tfrac{\theta}{2}\right)|1\rangle = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ e^{i\phi}\sin\left(\frac{\theta}{2}\right) \end{pmatrix}$$

  Mixed *states:*

$$\rho = \tfrac{1}{2}\left(\mathbb{1} + \vec{\boldsymbol{n}}\cdot\vec{\boldsymbol{\sigma}}\right) \qquad \text{s.t.:} \quad |\vec{\boldsymbol{n}}| \leqslant 1$$



Bloch ball

  - ***Unitary*** *transformations:* $\rho \longrightarrow U\rho U^\dagger$

  Each unitary has representation: $U = e^{i\alpha} R_{\vec{r}}(\vartheta)$

  where: $R_{\vec{r}}(\vartheta) \equiv \exp(-i\vartheta\,\vec{\boldsymbol{r}}\cdot\vec{\boldsymbol{\sigma}}/2) = \cos\left(\tfrac{\vartheta}{2}\right)\mathbb{1} - i\sin\left(\tfrac{\vartheta}{2}\right)\vec{\boldsymbol{r}}\cdot\vec{\boldsymbol{\sigma}}$

  - ***Measurement*** *in basis* $\{P_i\} = \{\,|\xi\rangle\langle\xi|\,,\,|\xi^\perp\rangle\langle\xi^\perp|\,\}$:

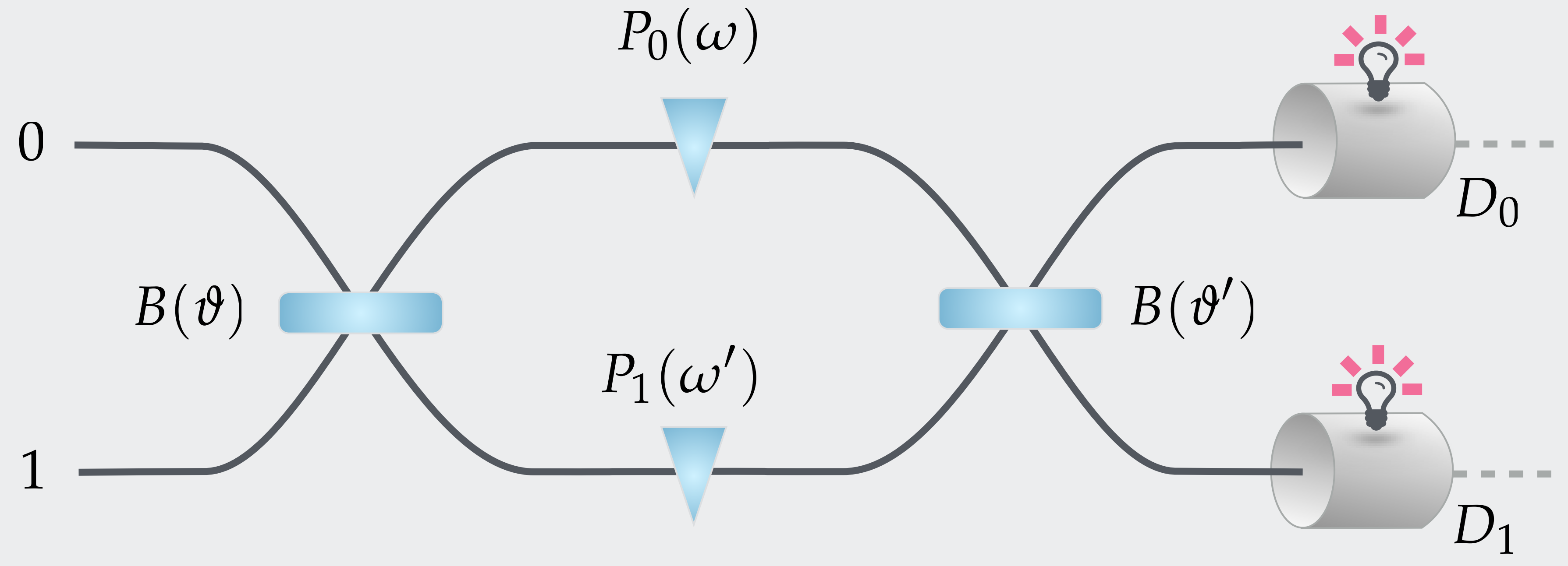
$$|\psi\rangle \longrightarrow \begin{cases} |\xi\rangle & \text{with: } P_0 = |\langle\xi|\psi\rangle|^2 \\ |\xi^\perp\rangle & \text{with: } P_1 = |\langle\xi^\perp|\psi\rangle|^2 \end{cases}$$

von Neumann projection + Born's rule

Quantum interferometry

Basic toolkit

- *Typical interferometric circuit:*
  - ***single-mode*** *and* ***one-particle*** *framework*
  - ***two paths*** *(spatially separated)*



$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad - \textit{ particle in path } \text{“}0\text{“}$$

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad - \textit{ particle in path } \text{“}1\text{“}$$

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

- ***Phase shifters:***

$$|\psi\rangle \xrightarrow{P_0(\omega)} \begin{pmatrix} e^{i\omega} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

$$|\psi\rangle \xrightarrow{P_1(\omega)} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\omega} \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

- ***Beam splitters:***

$$|\psi\rangle \xrightarrow{B(\vartheta)} \begin{pmatrix} i\cos\left(\frac{\vartheta}{2}\right) & \sin\left(\frac{\vartheta}{2}\right) \\ \sin\left(\frac{\vartheta}{2}\right) & i\cos\left(\frac{\vartheta}{2}\right) \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

- ***Detectors:***

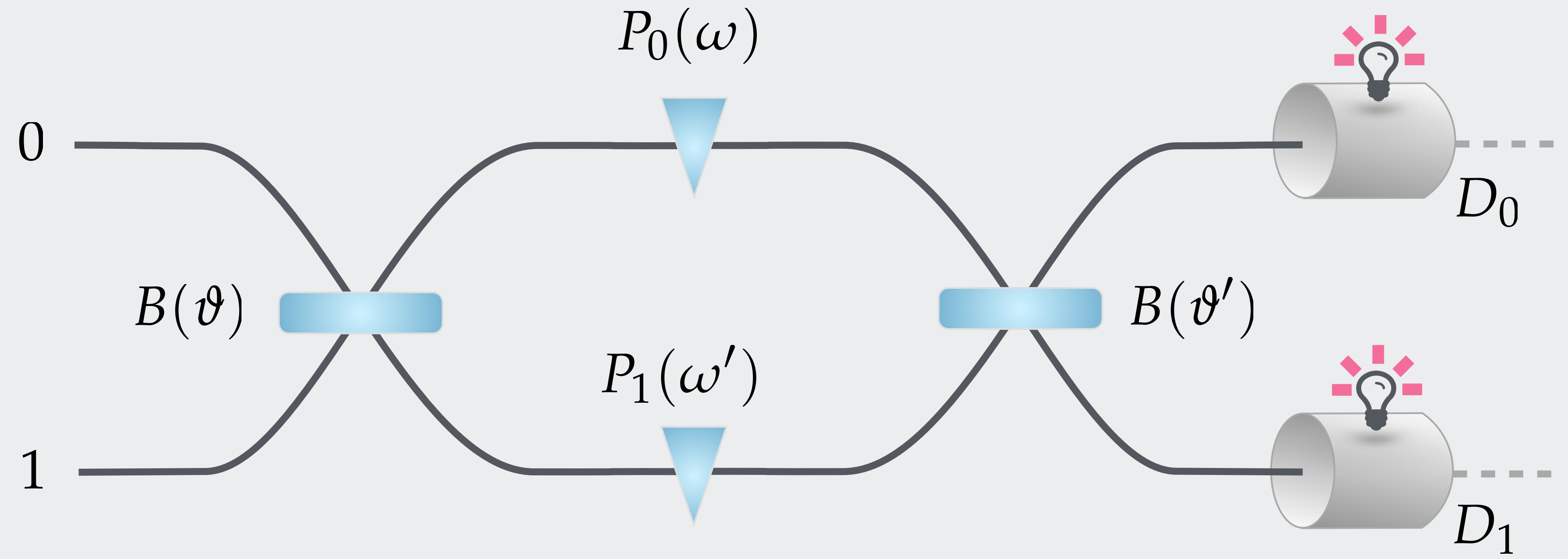
$$|\psi\rangle \xrightarrow{D_i} \begin{cases} |0\rangle & \text{with: } P_0 = |\alpha|^2 \\ |1\rangle & \text{with: } P_1 = |\beta|^2 \end{cases}$$

(*) *Mirror = phase shifter, path blocker = detector + post-selection*

Quantum interferometry

Basic toolkit

- *Typical interferometric circuit:*
  - ***single-mode*** *and* ***one-particle*** *framework*
  - ***two paths*** *(spatially separated)*

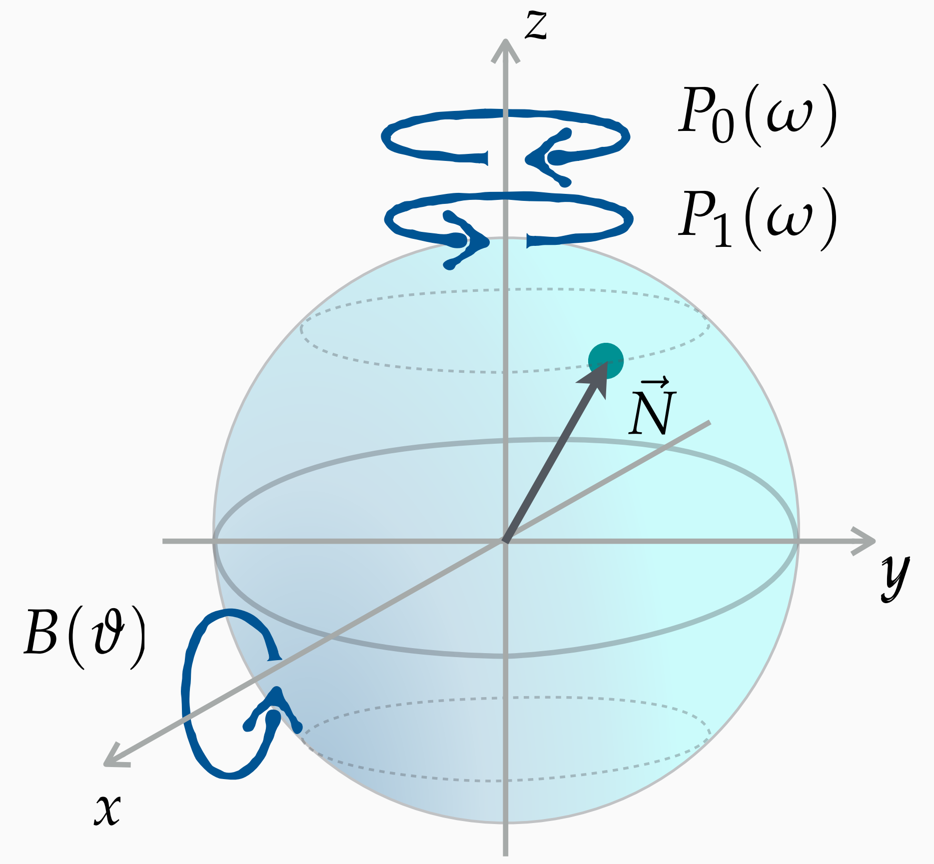


$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad - \text{ particle in path "0"}$$

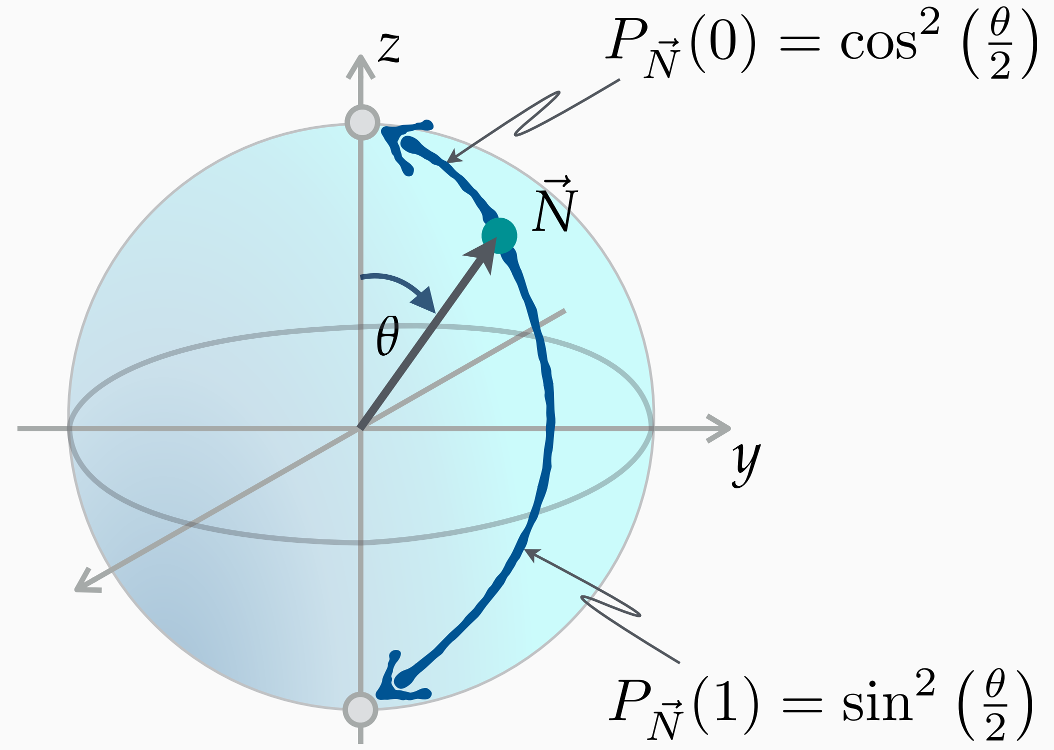
$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad - \text{ particle in path "1"}$$

$$|\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle = \begin{pmatrix} \alpha \\ \beta \end{pmatrix}$$

- ***Phase shifters:***
- ***Beam splitters:***



- ***Detectors:***



(*) *Mirror = phase shifter, path blocker = detector + post-selection*

Problems with the ontology

A few paradoxes for a qubit

Wave-particle duality

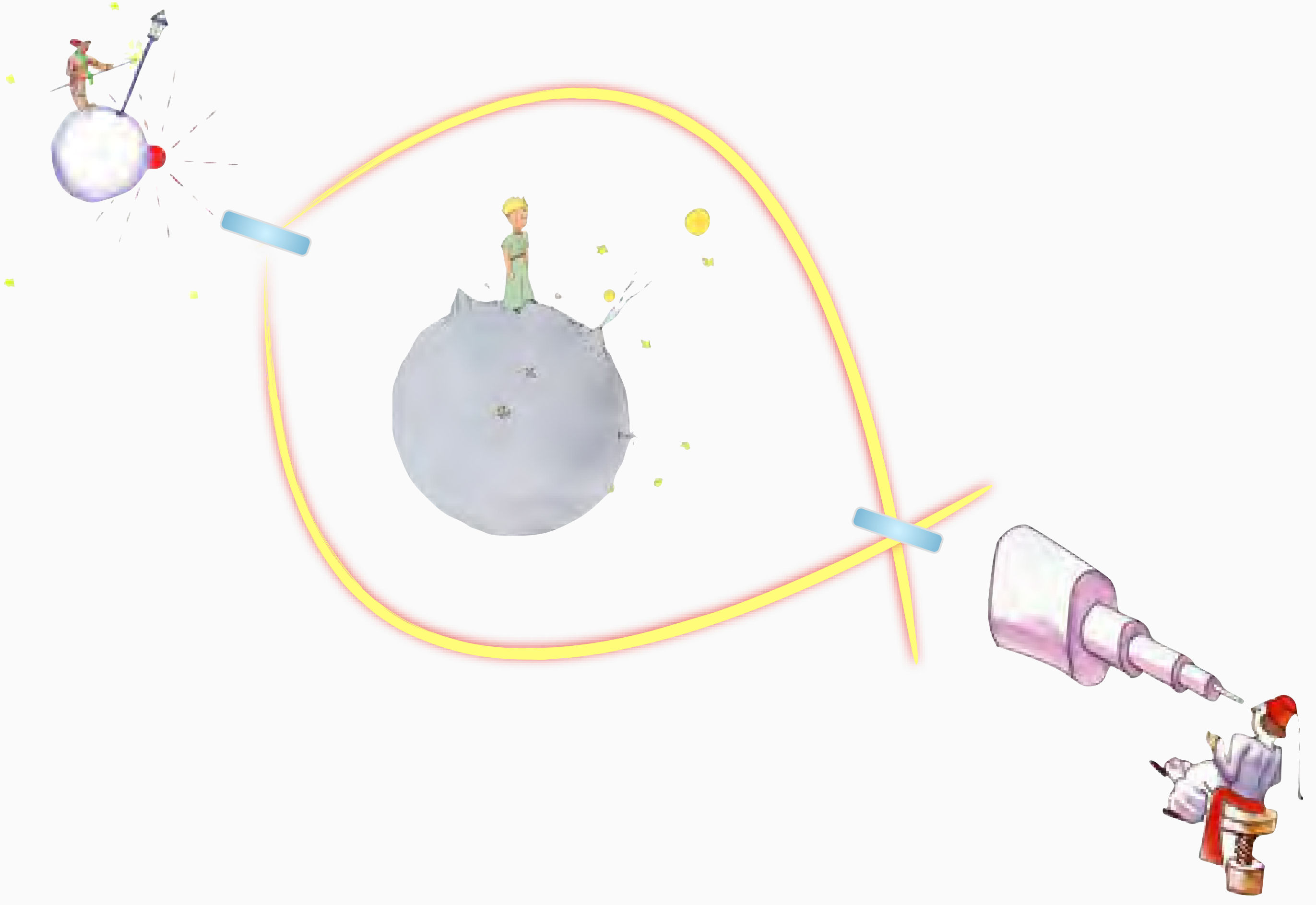
Wheeler's delayed-choice experiment

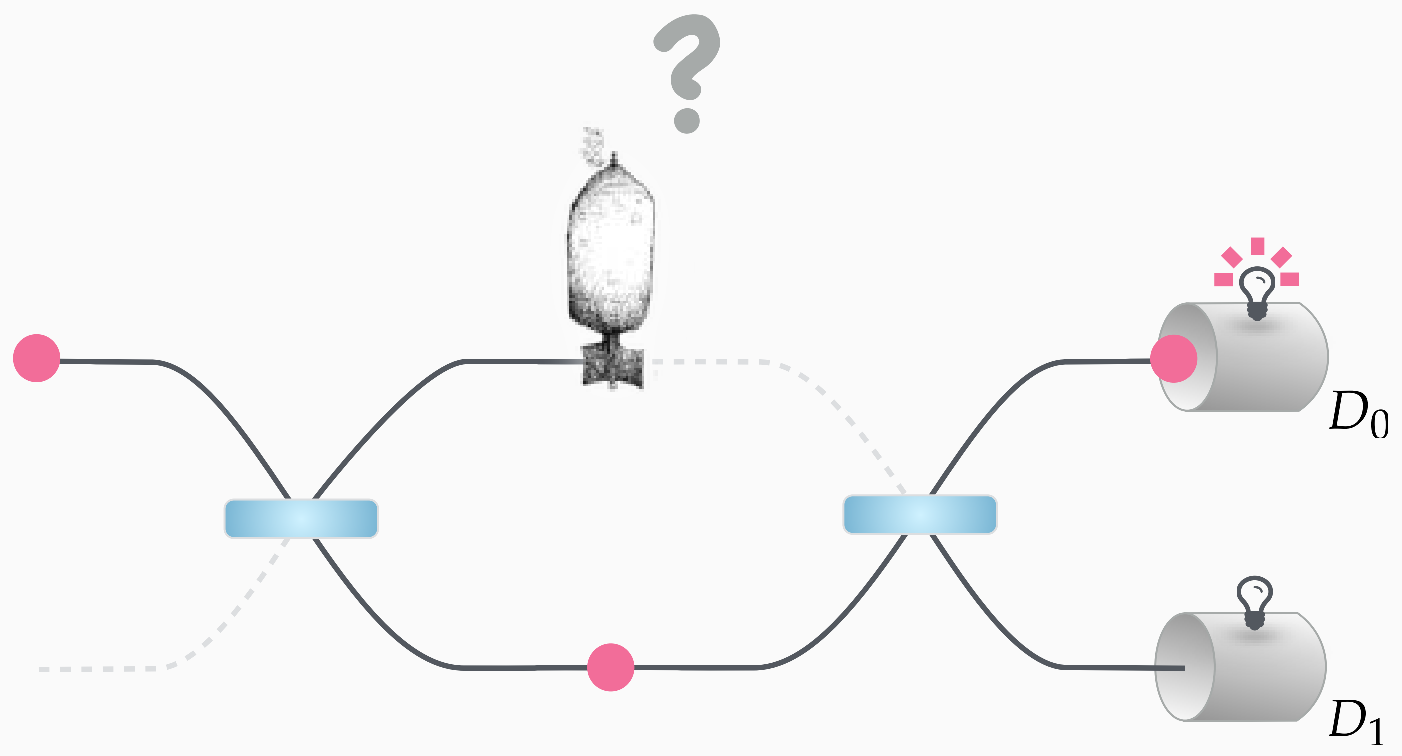
Non-locality and interaction-free measurements

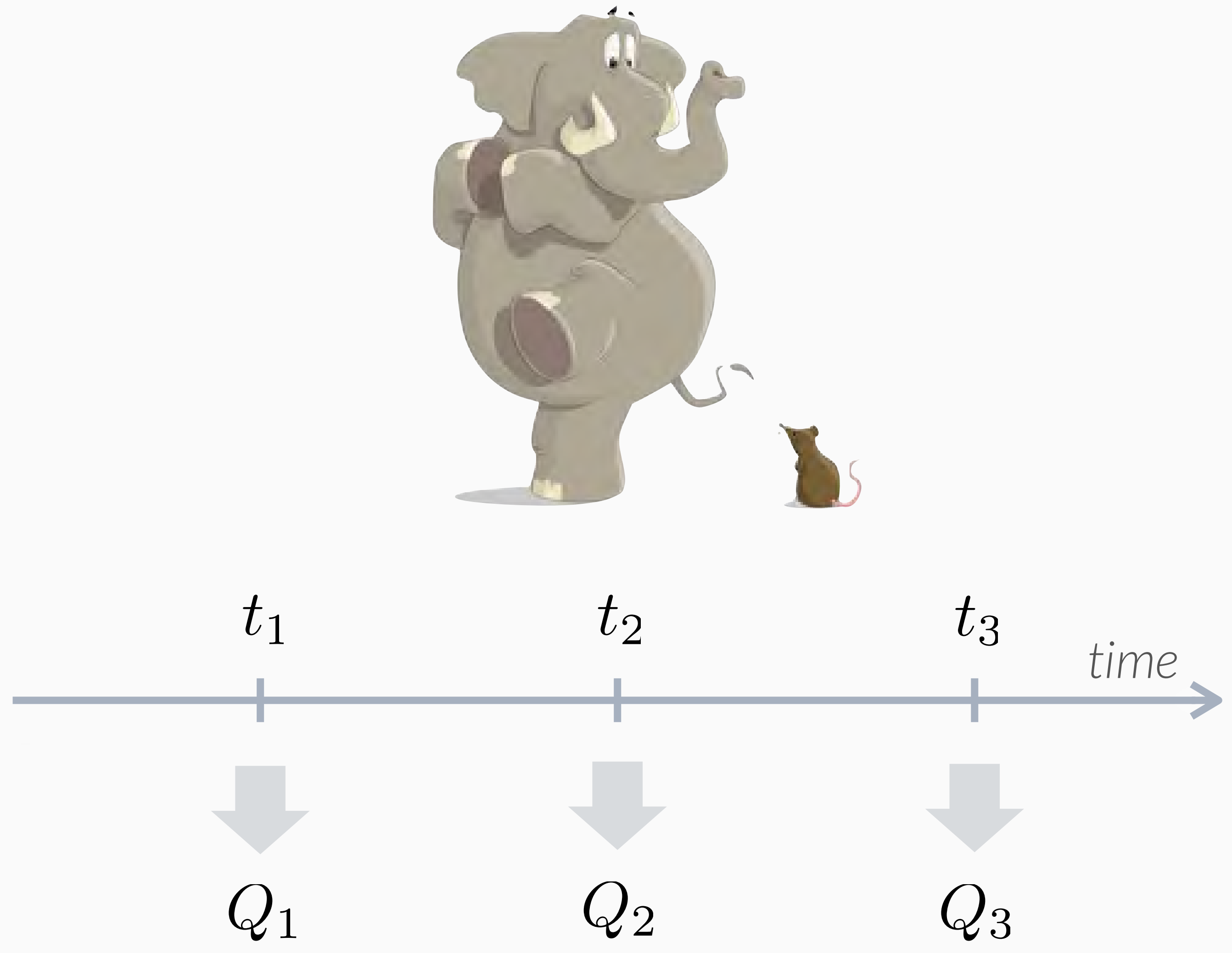
Elitzur-Vaidman bomb testing problem

Micro vs. macroscopic realism

Leggett-Garg inequalities







*How the particle particle '**changes**' its past ?*

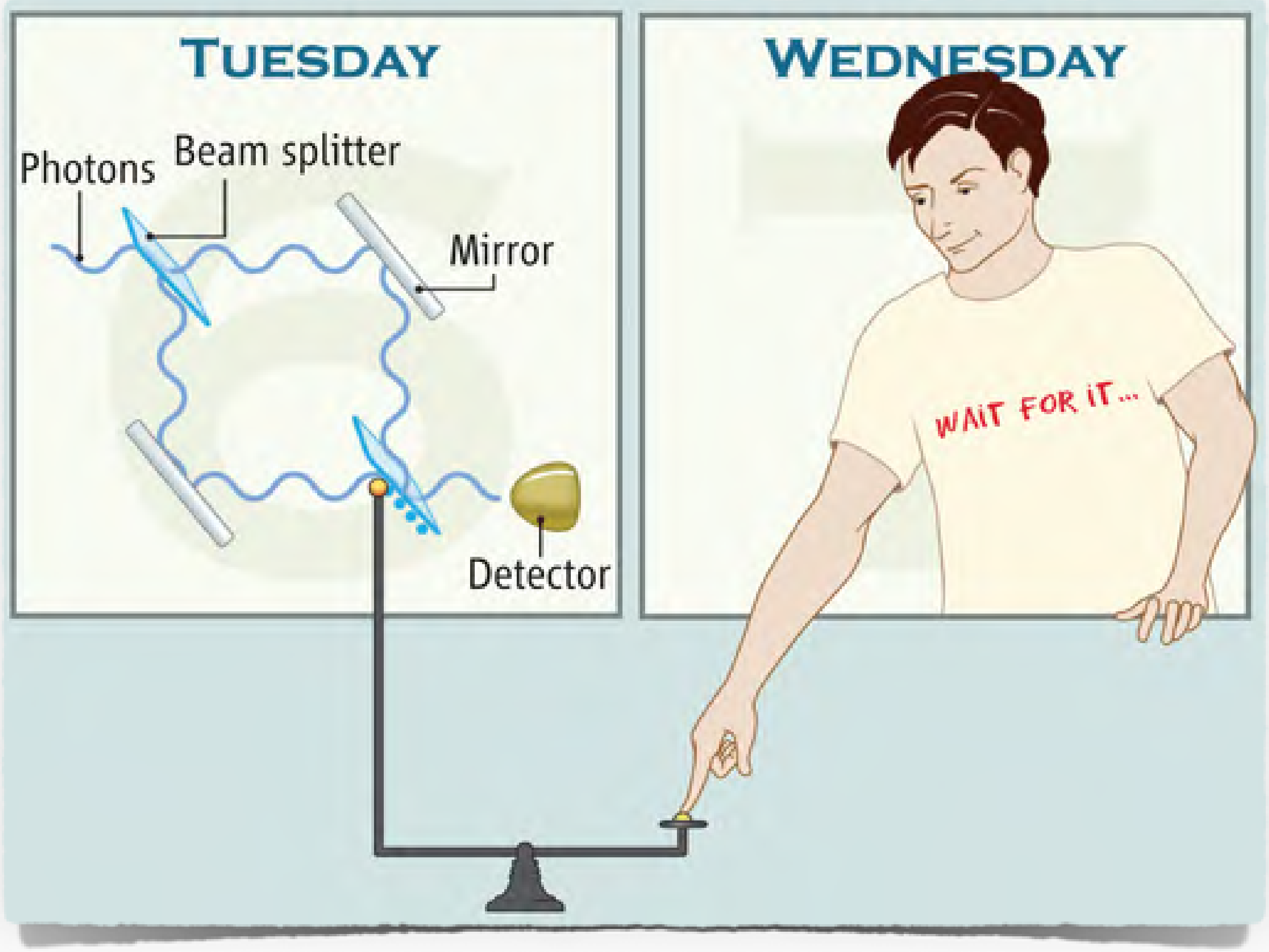
*How the particle '**feels**' the other path ?*

*How the world becomes '**macro**' ?*

Problems with the ontology

A few paradoxes for a qubit

- ***Wave-particle duality***
  Wheeler's delayed-choice experiment



PHYSICS

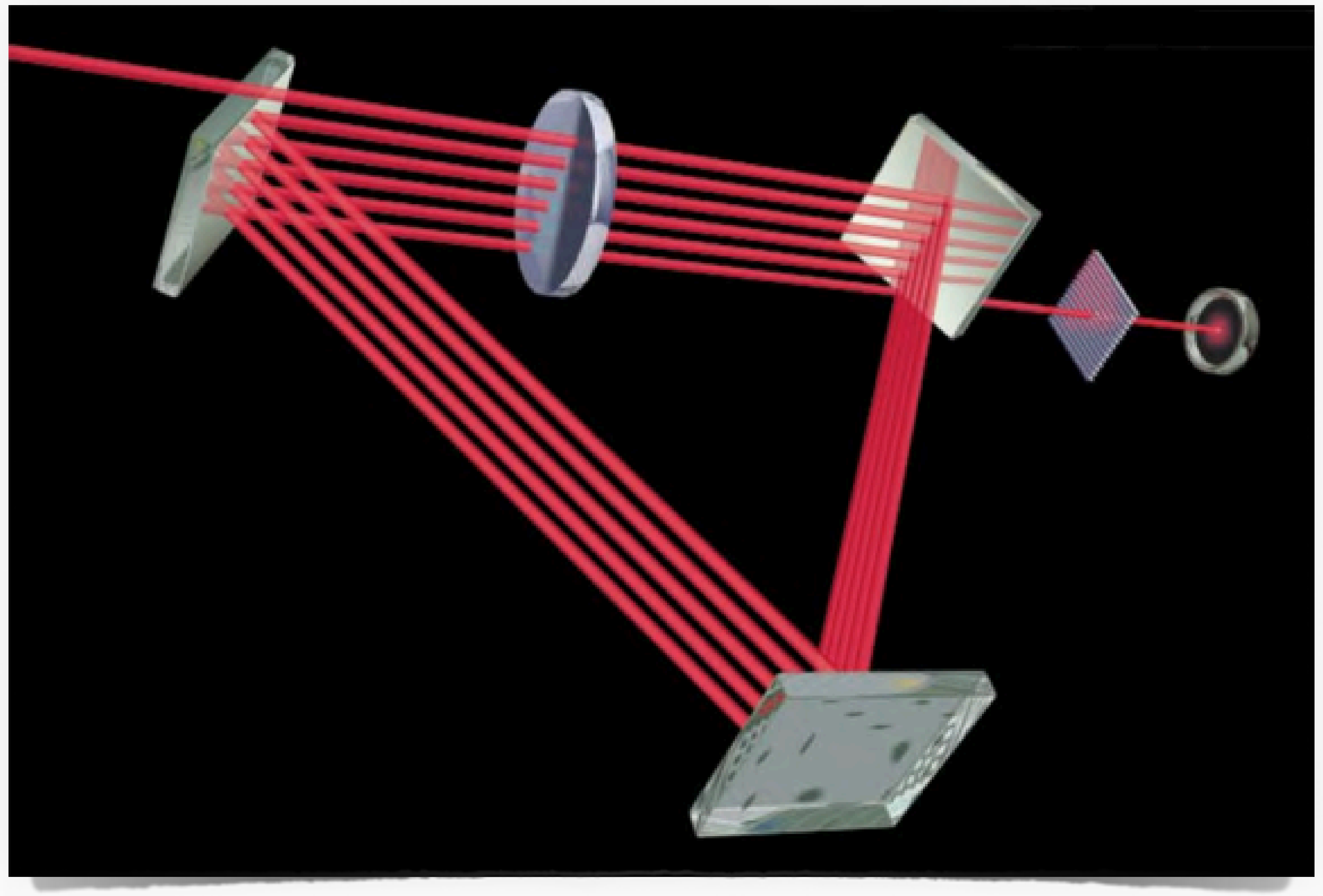
Quantum Procrastination

Seth Lloyd

Entangling two photons allows the wave and particle nature of light to be interchanged even after the light has already been detected.

Science **338** 621 (2012)

- ***Non-locality and interaction-free measurements***
  Elitzur-Vaidman bomb testing problem



Quantum Seeing in the Dark

Quantum optics demonstrates the existence of interaction-free measurements: the detection of objects without light—or anything else—ever hitting them

by Paul Kwiat, Harald Weinfurter and Anton Zeilinger

Scientific American **275** 72 (1996)

- ***Micro vs. macroscopic realism***
  Leggett-Garg inequalities

QUANTUM MECHANICS

No moon there

An experiment reveals that micrometre-sized superconducting circuits follow the laws of quantum mechanics, and thus defy common experience of how macroscopic objects should behave.

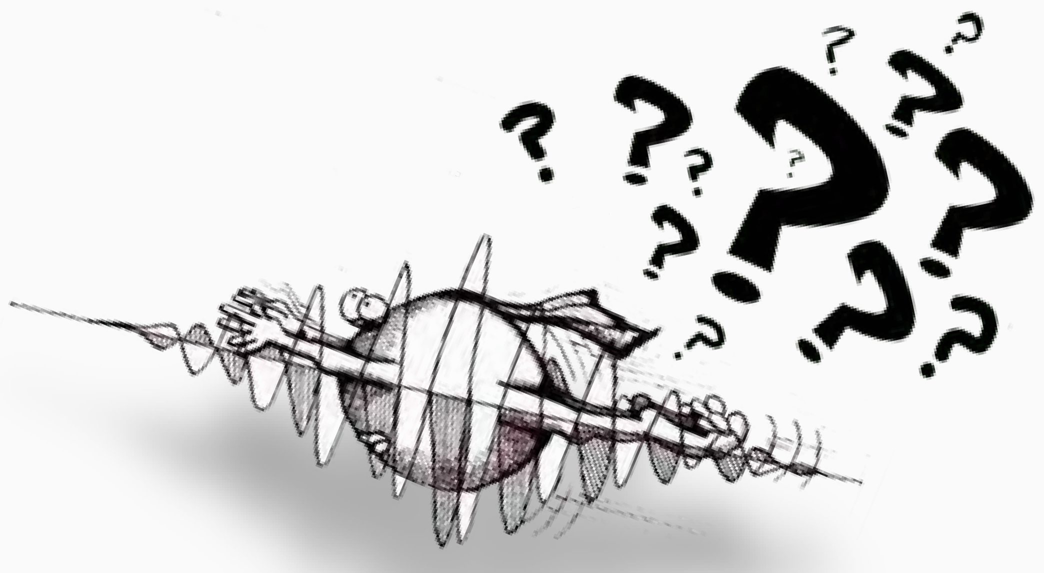
Johan E. Mooij

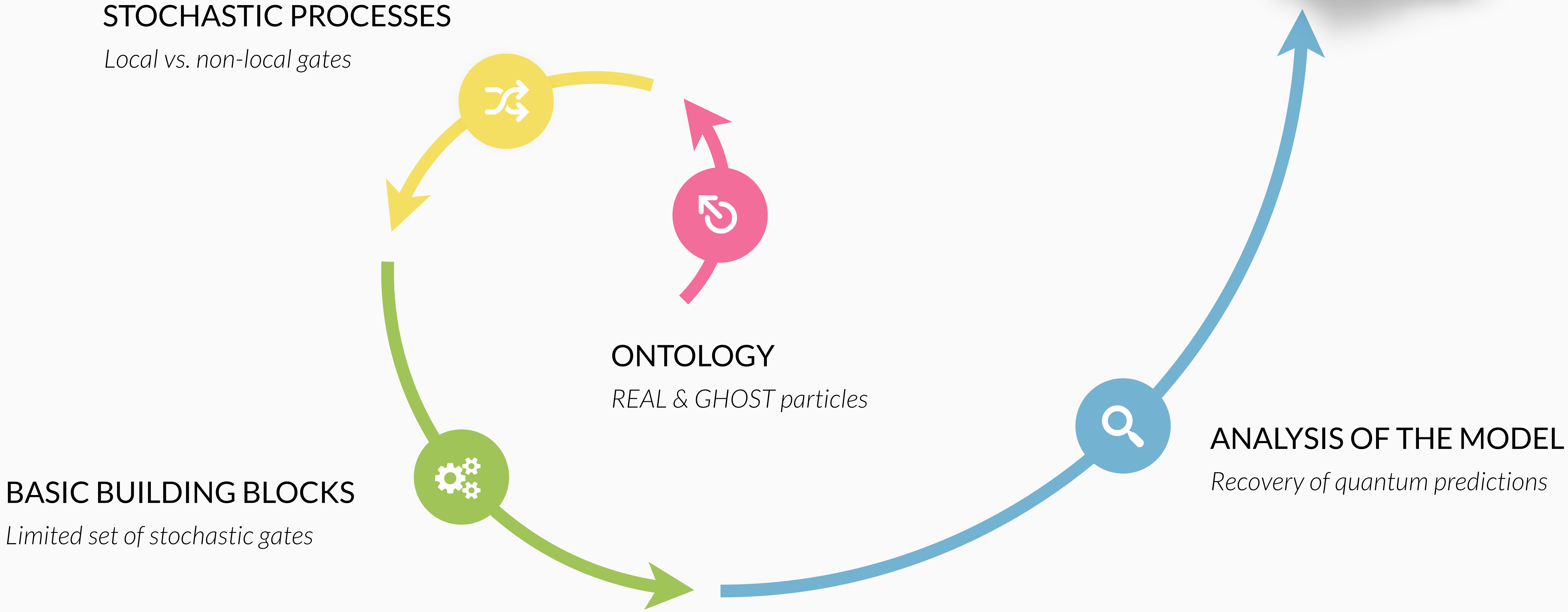
Nature Physics **6** 401 (2010)

Building the model

Plan of action

*Is it possible to make sense of interferometric experiments with a qubit in '**classical**' terms ?*
*Can you see it as a **stochastic process** ? Do **correlations** help ? What about **locality** ?*





Reminder I

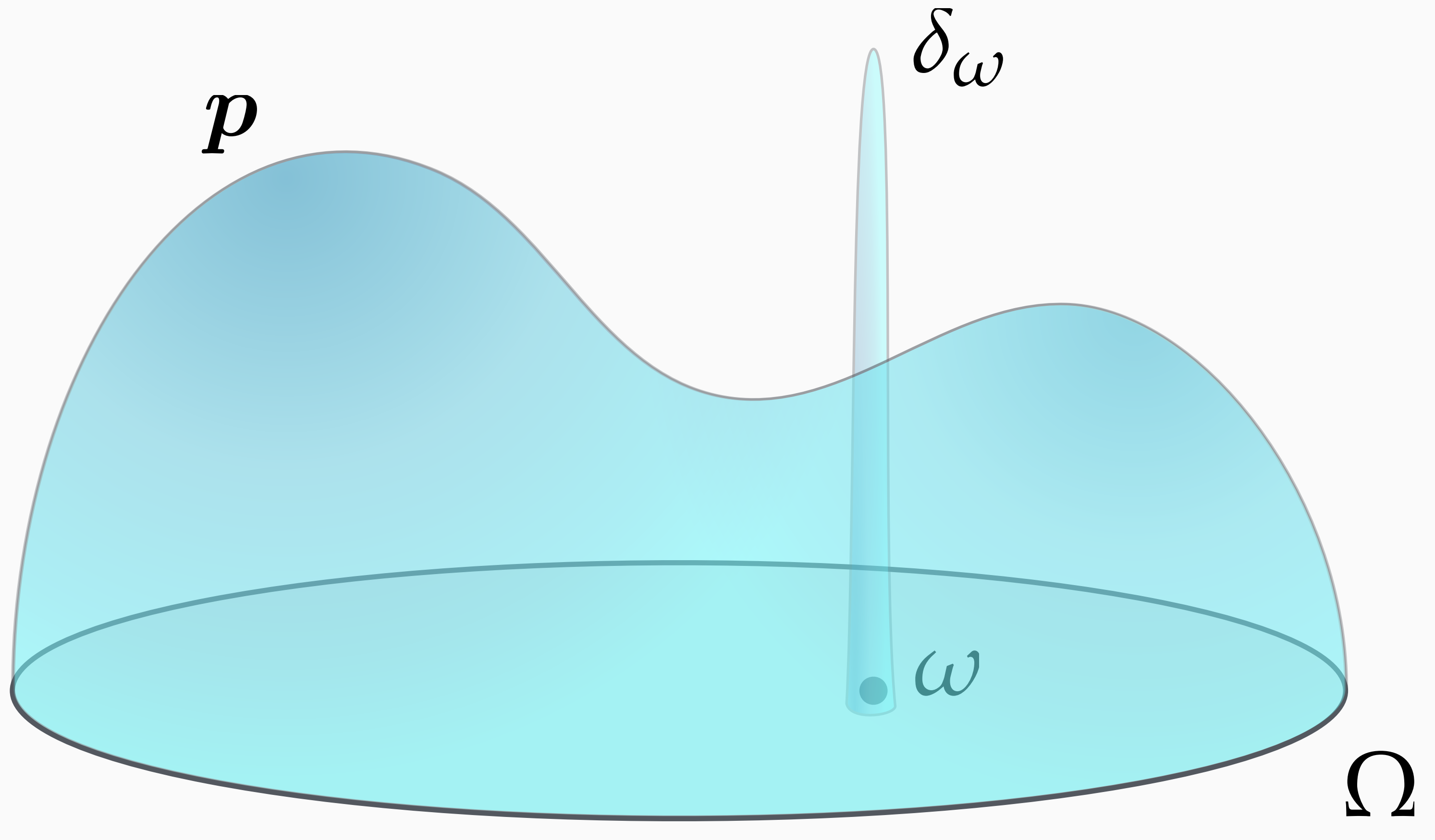
Probabilistic set-up

- Ontic state space: Ω

  Probabilistic description: $\mathcal{P}(\Omega) = \left\{ \boldsymbol{p} : \Omega \longrightarrow [0,1] \ : \ \sum_{\omega} p(\omega) = 1 \right\}$

  Ontic states: $\omega \ \leftrightsquigarrow \ \boldsymbol{p} = \delta_\omega$

  In general: $\boldsymbol{p} = \sum_{\omega} p(\omega)\, \delta_\omega$

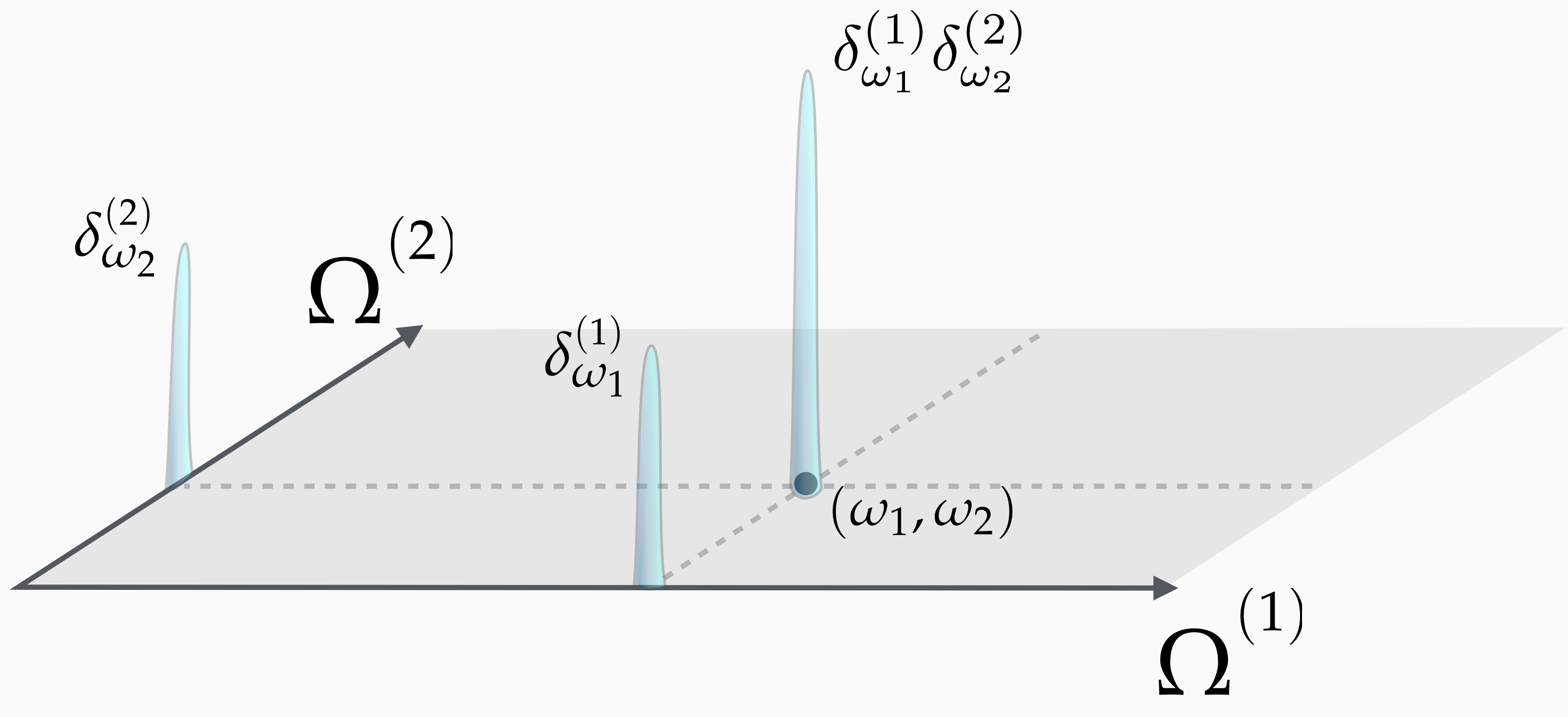


- Product ontic state space: $\Omega = \Omega^{(1)} \times \Omega^{(2)}$

  Probabilistic description: $\mathcal{P}(\Omega) = \mathcal{P}\left(\Omega^{(1)}\right) \otimes \mathcal{P}\left(\Omega^{(2)}\right)$

  Ontic states: $(\omega_1, \omega_2) \ \leftrightsquigarrow \ \boldsymbol{p} = \delta^{(1)}_{\omega_1} \otimes \delta^{(2)}_{\omega_1} = \delta^{(1)}_{\omega_1}\, \delta^{(2)}_{\omega_2}$

  In general: $\boldsymbol{p} = \sum_{\omega_1, \omega_2} p(\omega_1, \omega_2)\, \delta^{(1)}_{\omega_1}\, \delta^{(2)}_{\omega_2}$



Reminder II

Stochastic transformations

- *Deterministic:* $T : \Omega \longrightarrow \Omega$
- *Probabilistic*[*]: $T : \Omega \longrightarrow \mathcal{P}(\Omega)$

Conditional probabilities:

Stochastic matrix

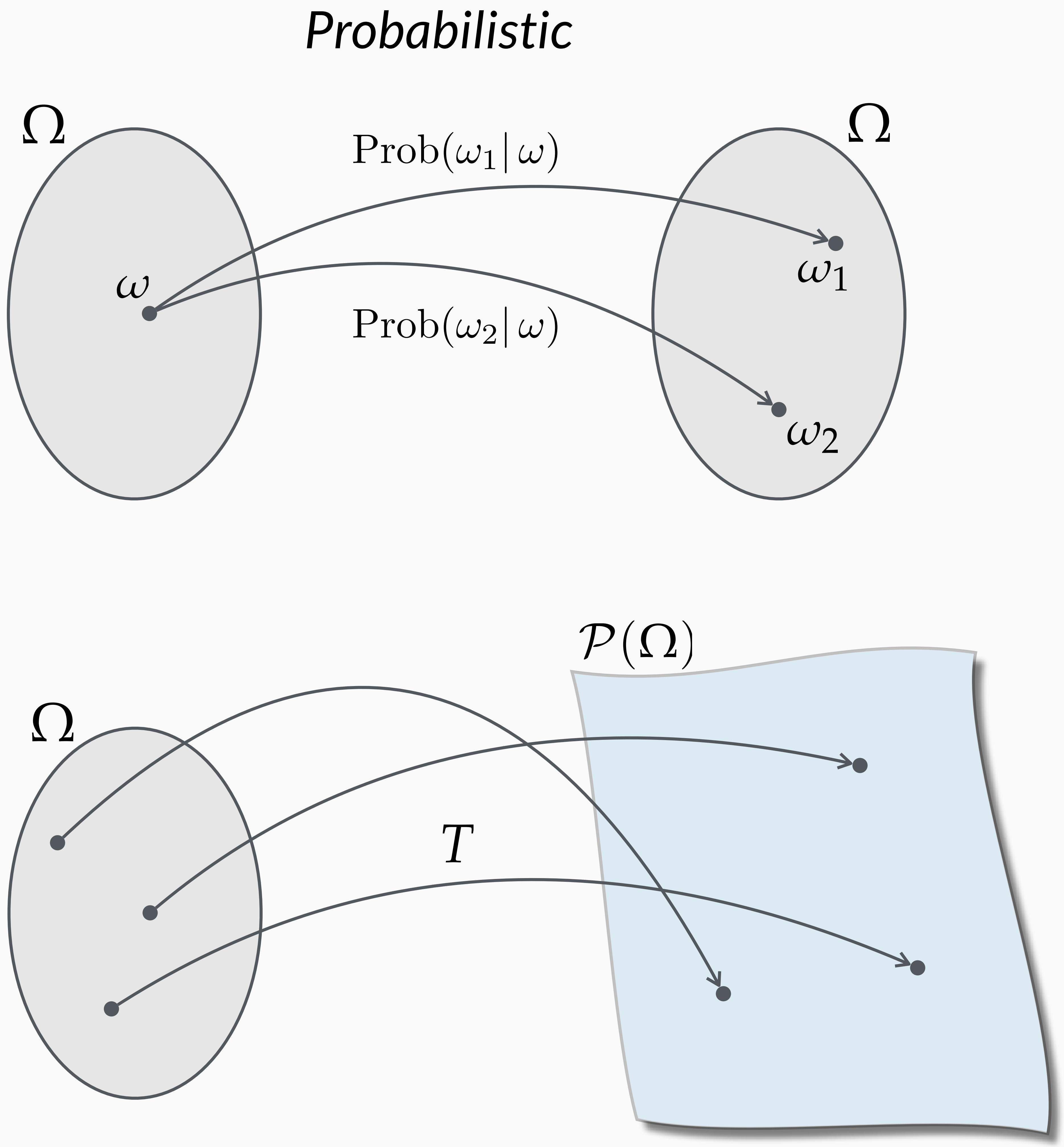
$$T(i)(k) = \mathrm{Prob}(\omega = k | \omega = i) = \mathbb{T}_{ki}$$

Then: $\boldsymbol{p} \xrightarrow{T} \boldsymbol{p}' = \mathbb{T}\boldsymbol{p}$

$$p'(k) = \sum_i \mathrm{Prob}(\omega = k | \omega = i)\, p(i)$$

For a sequence : $\boldsymbol{p}_0 \xrightarrow{T_1} \boldsymbol{p}_1 \xrightarrow{T_2} \boldsymbol{p}_2 \xrightarrow{T_3} \boldsymbol{p}_3$

we have: $\boldsymbol{p}_3 = \mathbb{T}_3\, \mathbb{T}_2\, \mathbb{T}_1\, \boldsymbol{p}_0$



[*] *Compare with:* $T : \mathcal{P}(\Omega) \longrightarrow \mathcal{P}(\Omega)$

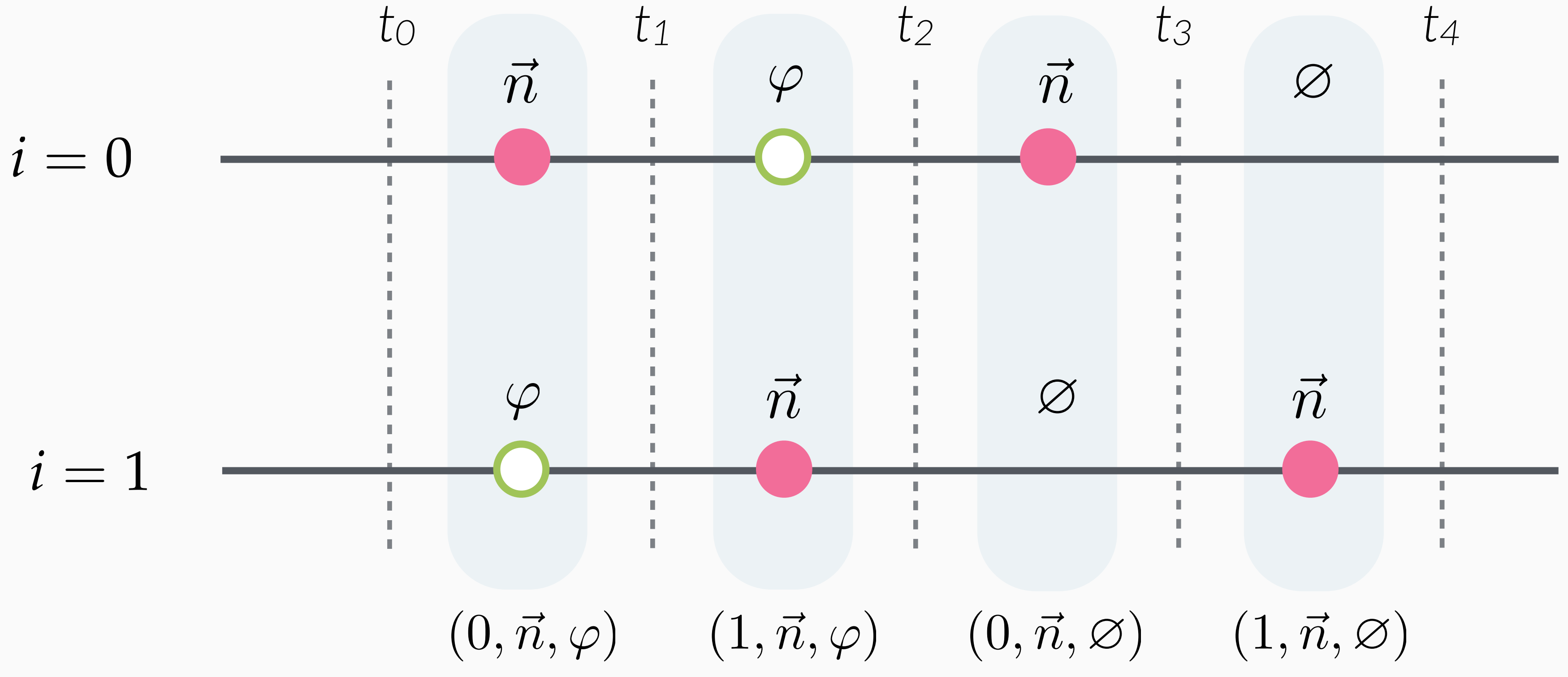
Ontology of the Model

General set-up: Ontic state space

- *Two paths:* $i = 0, 1$
- *Two kinds of particles:*
  - *REAL particles:* $\vec{n} = (\theta, \phi) \in S^2$
  - *GHOST particles:* $\varphi \in S^1$
- ***Key assumption:***

  *Only **single** REAL particle present in the circuit,*

  *with a GHOST in the **other** path or the path is EMPTY.*



*Hence, the **ontic state** space:*

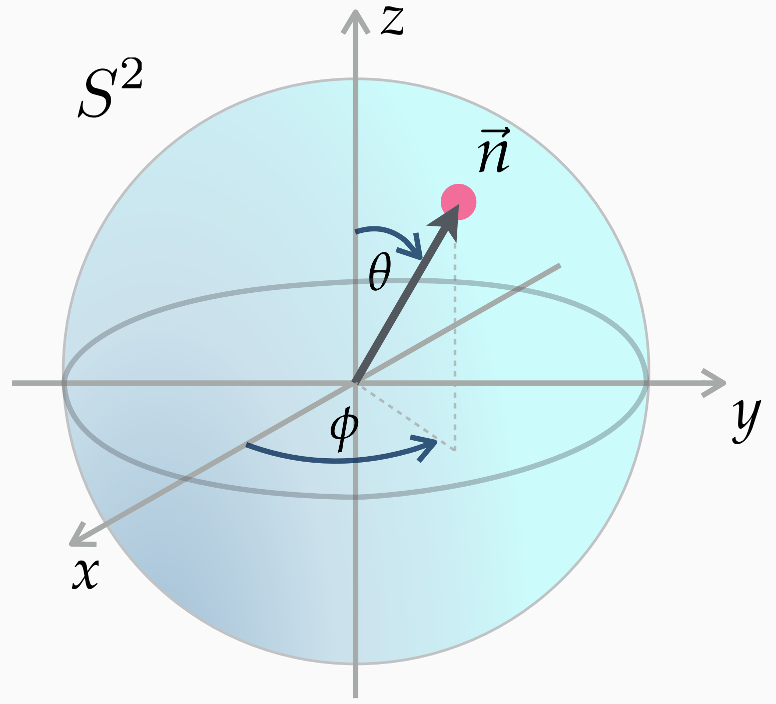
$$\Omega \equiv \{0, 1\} \times S^2 \times S^{1^*} \ni (i, \vec{n}, \varphi) \ \text{ or } \ (i, \vec{n}, \varnothing)$$

where is REAL particle

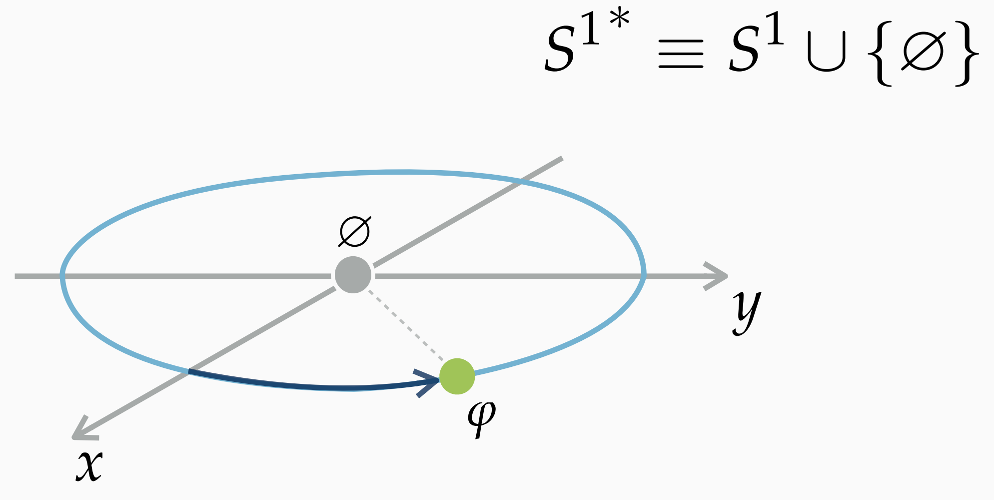
inner state of REAL particle

inner state of GHOST particle or EMPTY

$S^{1^*} \equiv S^1 \cup \{\varnothing\}$



REAL



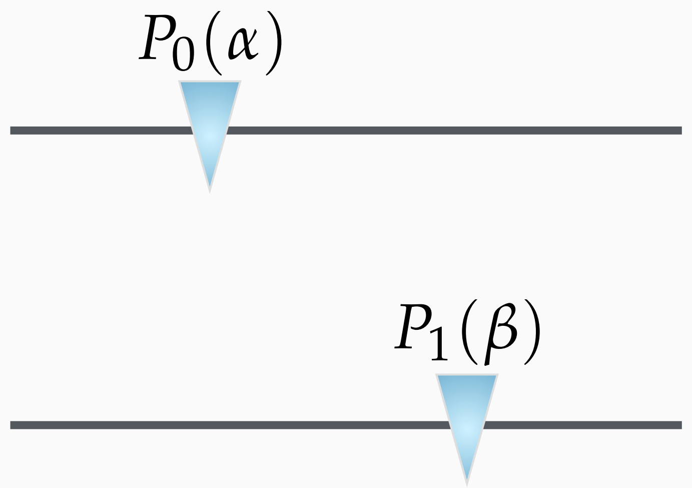
GHOST / EMPTY

Building Blocks of the Model

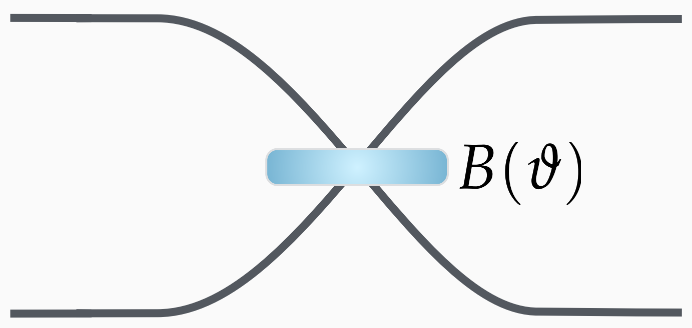
Limited set of stochastic gates

*We will consider **stochastic circuits** that are built from a few **building blocks**:*

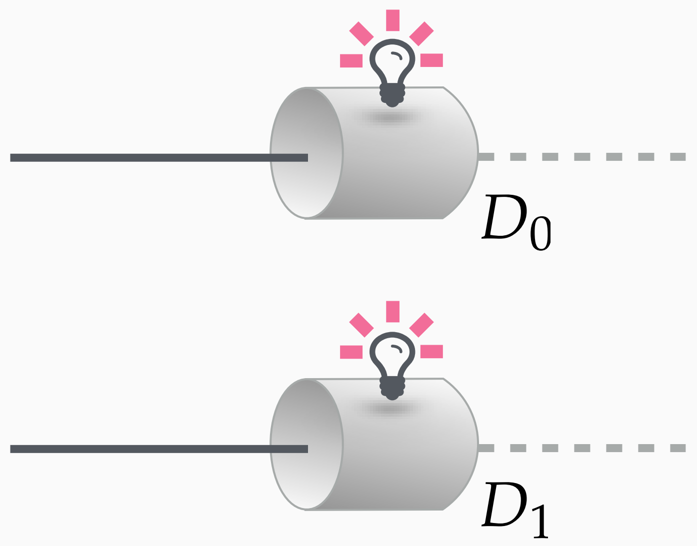
- ***Phase shifters:***



- ***Beam splitters:***



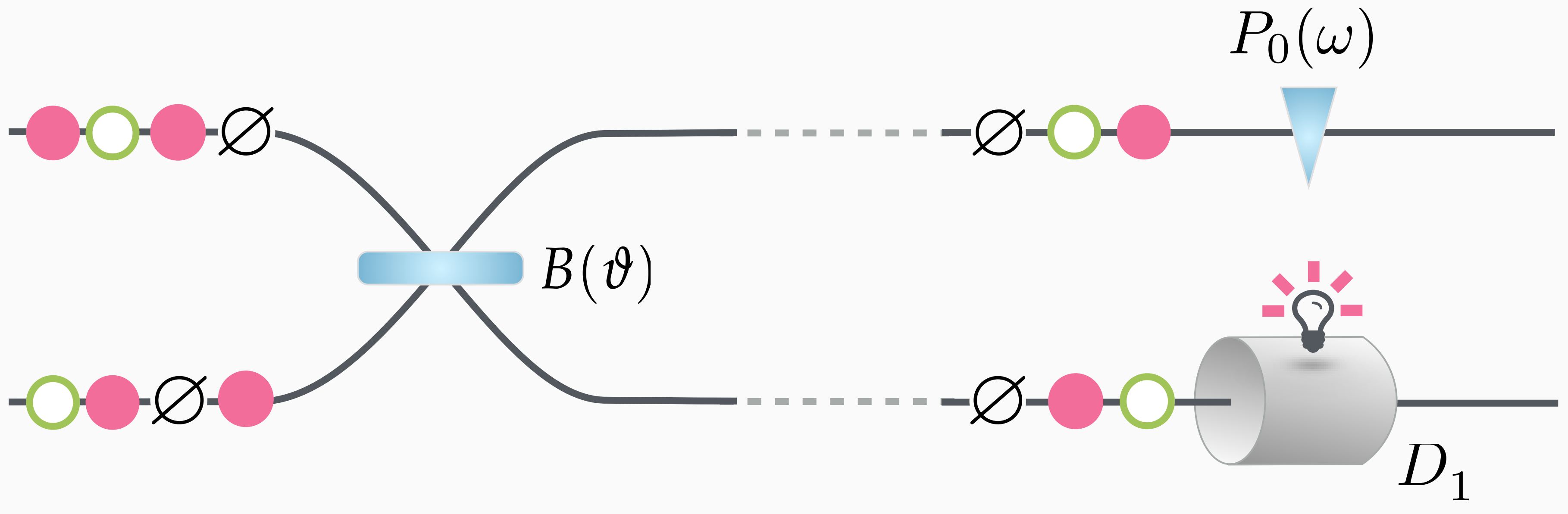
- ***Detectors:***



- *Need to specify how they act the **ontic states**.*
- *Make sure that **phase shifters** and **detectors** act locally and only **beam splitter** has access to both paths.*

REAL & GHOST : $(i, \vec{n}, \varphi) \longrightarrow \boldsymbol{p} \in \mathcal{P}(\Omega)$

REAL & EMPTY: $(i, \vec{n}, \varnothing) \longrightarrow \boldsymbol{p} \in \mathcal{P}(\Omega)$



Phase shifter

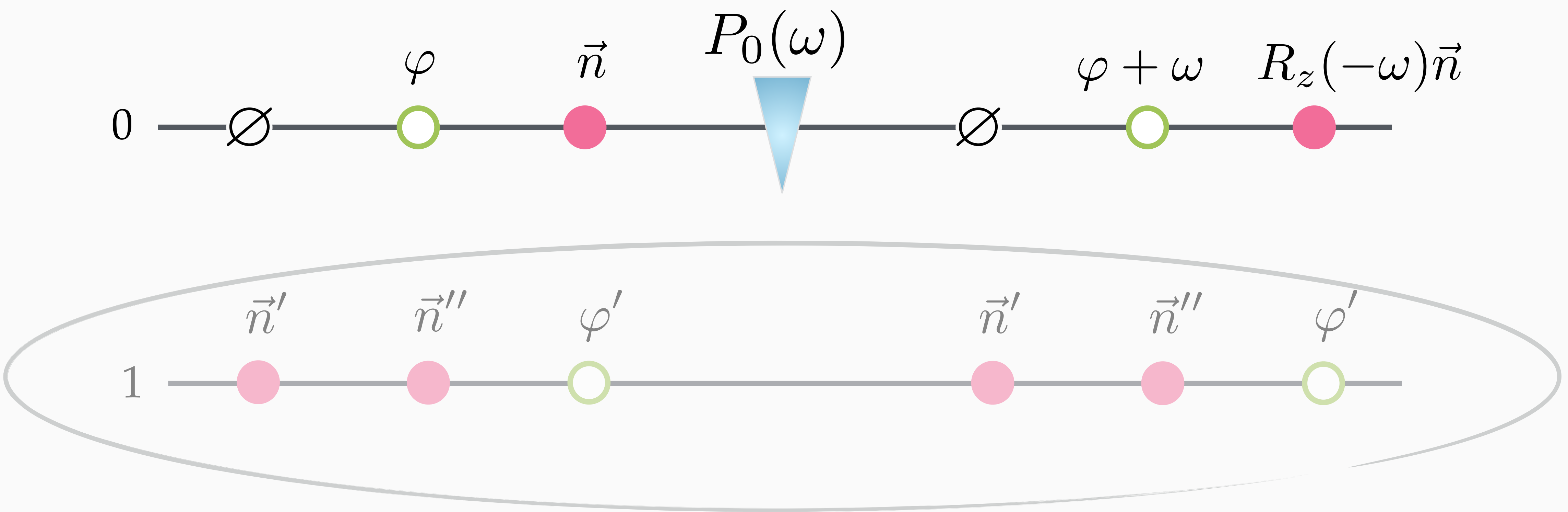
Action of **phase shifter** $P_0(\omega)$ in the 0-th path:

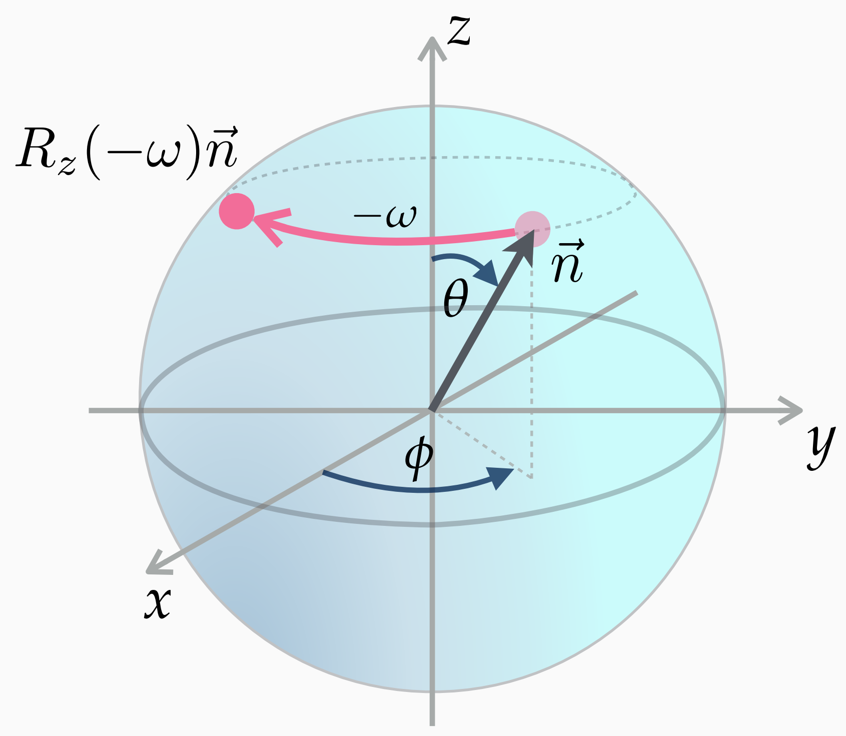
- rotates REAL particle around $\hat{z}$ axis by $-\omega$,
- rotates GHOST particle around $\hat{z}$ axis by $+\omega$,
- for EMPTY $\varnothing$ does nothing.

$$(i, \vec{n}, \varphi) \xrightarrow{P_0(\omega)} \begin{cases} \delta_0\, \delta_{R_z(-\omega)\,\vec{n}}\, \delta_\varphi & \text{if} \quad i = 0\,, \\ \delta_1\, \delta_{\vec{n}}\, \delta_{\varphi+\omega} & \text{if} \quad i = 1\,. \end{cases}$$

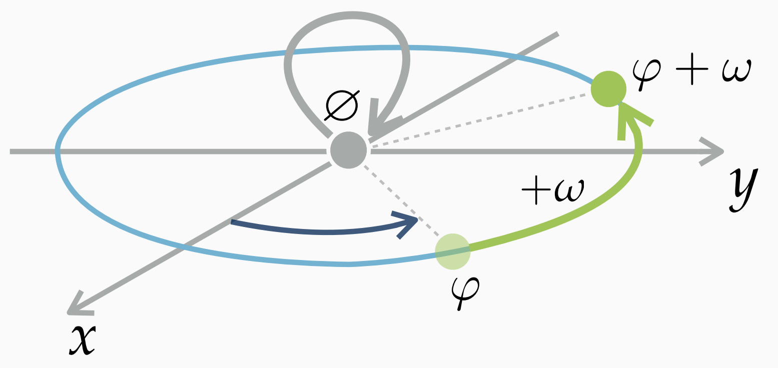
$$(i, \vec{n}, \varnothing) \xrightarrow{P_0(\omega)} \begin{cases} \delta_0\, \delta_{R_z(-\omega)\,\vec{n}}\, \delta_\varnothing & \text{if} \quad i = 0\,, \\ \delta_1\, \delta_{\vec{n}}\, \delta_\varnothing & \text{if} \quad i = 1\,. \end{cases}$$

Local deterministic gate !!





REAL
$(i = 0)$



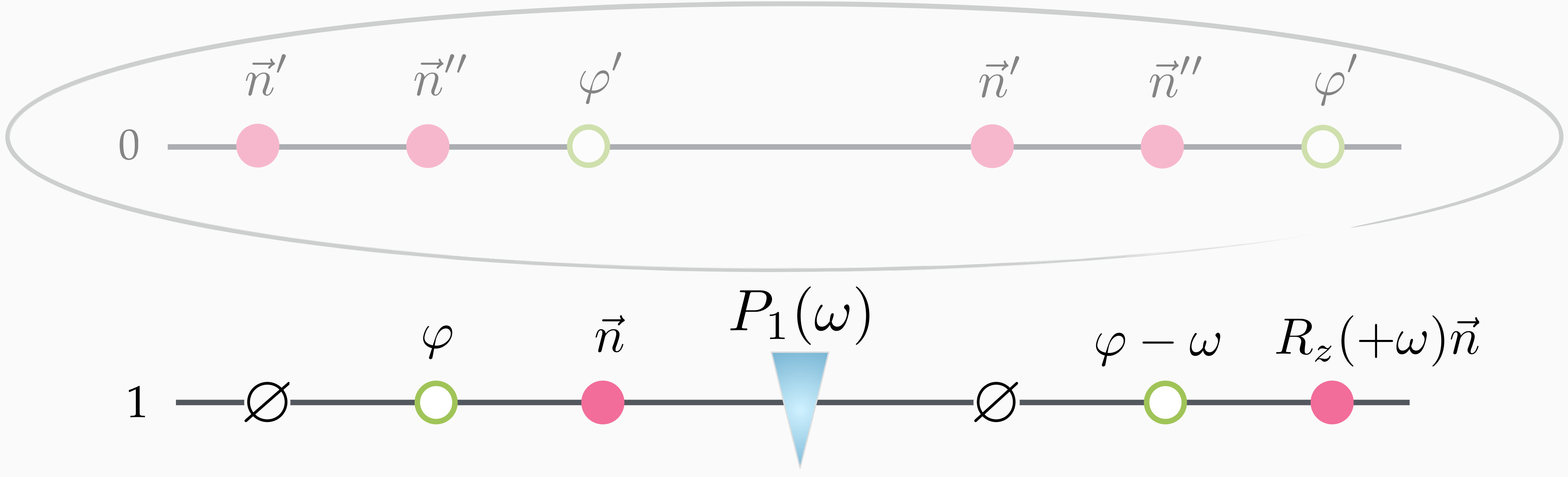
GHOST / EMPTY
$(i = 1)$

Phase shifter

Action of ***phase shifter*** $P_1(\omega)$ in the 1-th path:

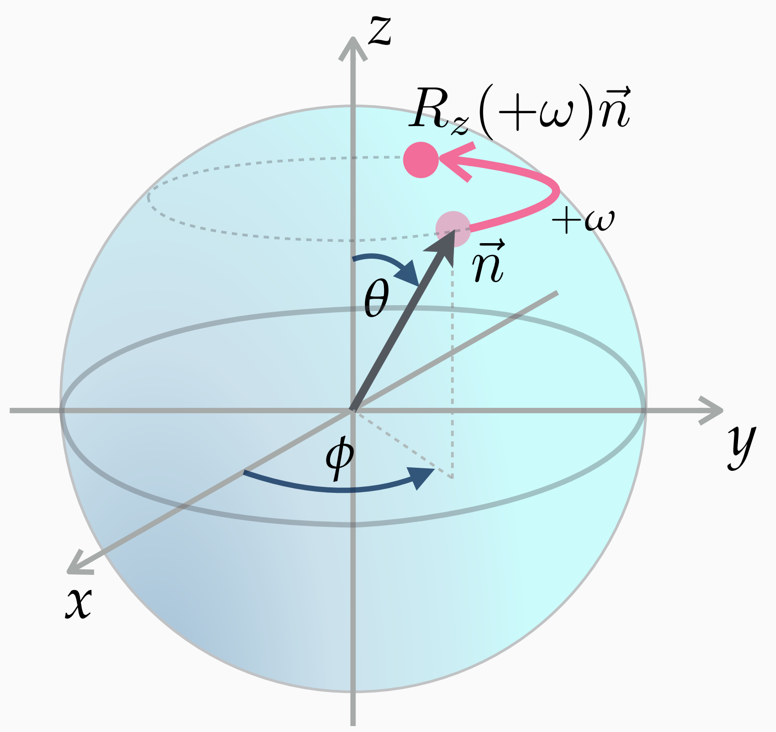
- rotates REAL particle around $\hat{z}$ axis by $+\omega$,
- rotates GHOST particle around $\hat{z}$ axis by $-\omega$,
- for EMPTY $\varnothing$ does nothing.

Local deterministic gate !!

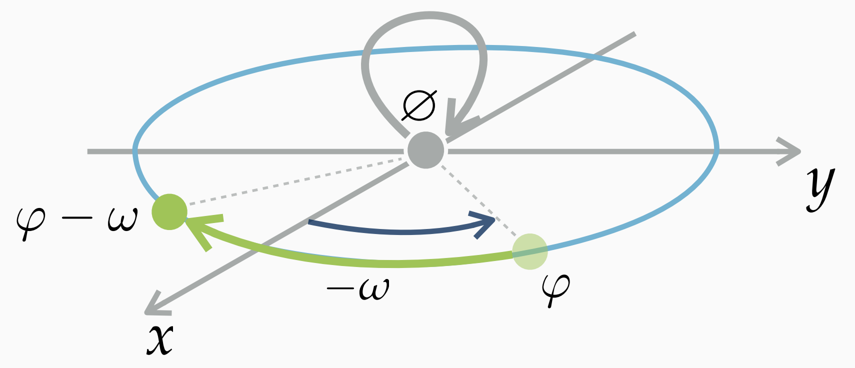


$$(i,\vec{n},\varphi) \xrightarrow{P_1(\omega)} \begin{cases} \delta_0\, \delta_{\vec{n}}\, \delta_{\varphi-\omega} & \text{if} \quad i=0\,, \\ \delta_1\, \delta_{R_z(\omega)\,\vec{n}}\, \delta_{\varphi} & \text{if} \quad i=1\,. \end{cases}$$

$$(i,\vec{n},\varnothing) \xrightarrow{P_1(\omega)} \begin{cases} \delta_0\, \delta_{\vec{n}}\, \delta_{\varnothing} & \text{if} \quad i=0\,, \\ \delta_1\, \delta_{R_z(\omega)\,\vec{n}}\, \delta_{\varnothing} & \text{if} \quad i=1\,. \end{cases}$$



REAL
$(i=0)$



GHOST / EMPTY
$(i=1)$

(Detector)

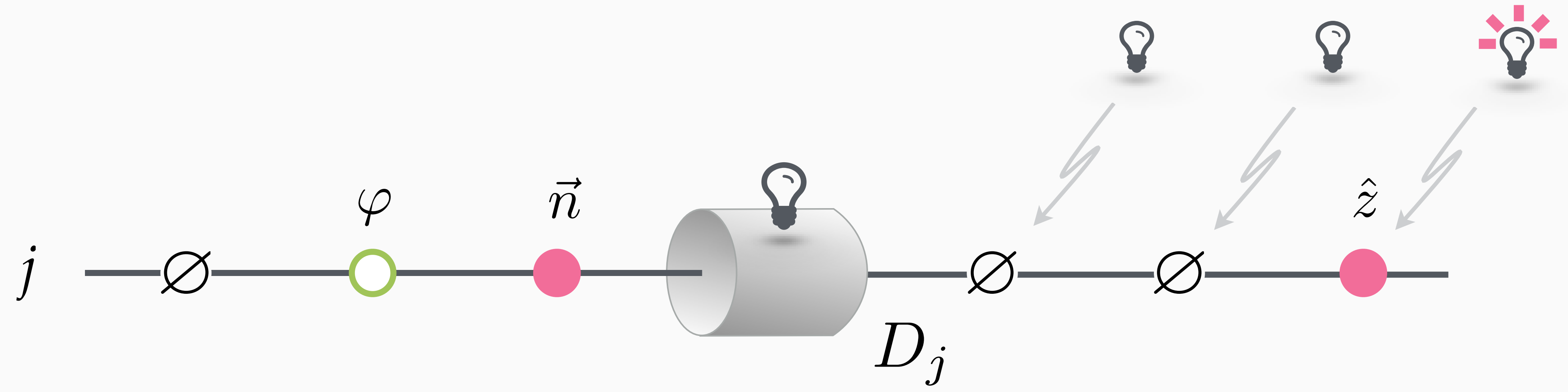
Action of **detector** D_j in the j-th path:

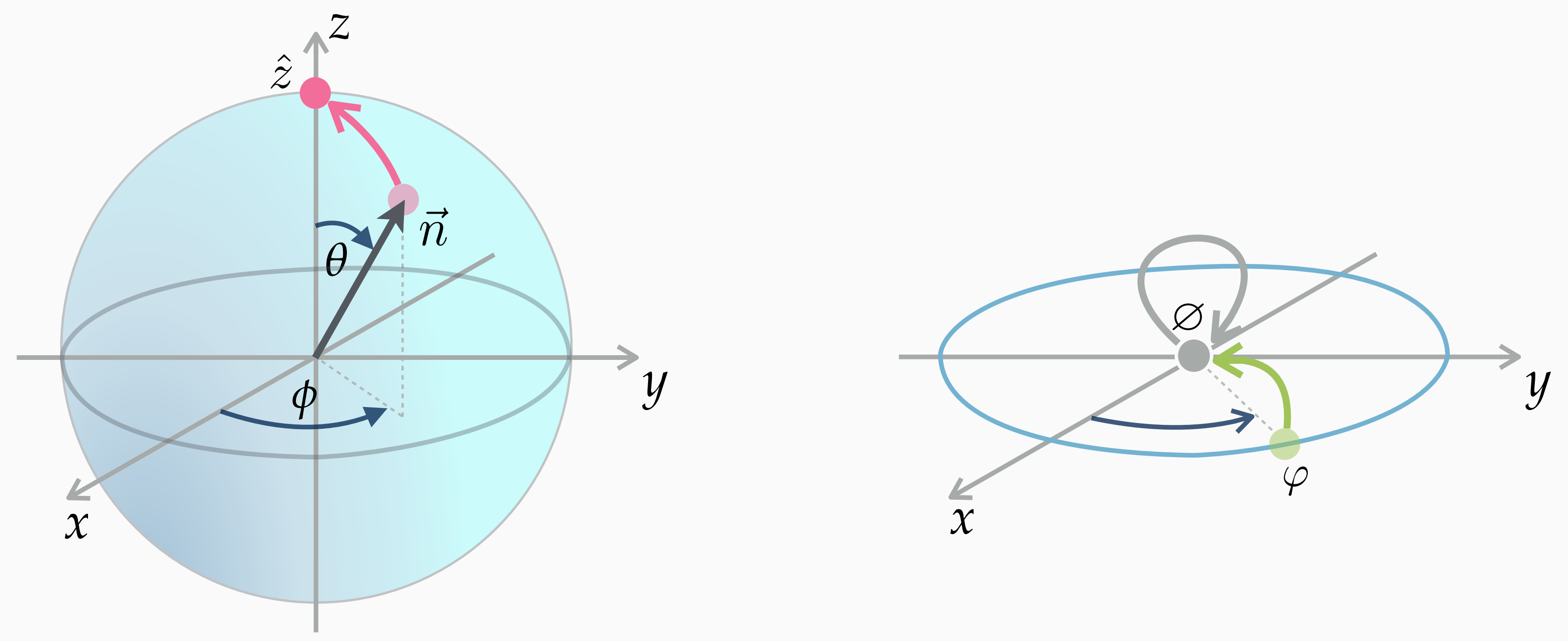
- ***reveals*** ('clicks') whether REAL particle is in j-th path and if ***yes*** leaves it in state $\vec{n} \to \hat{z}$,
- remains ***silent*** ('no click') about the GHOST and ***removes*** it from the channel,
- for EMPTY $\varnothing$ does nothing ('no click').

$$(i, \vec{n}, \varphi) \xrightarrow{D_j} \begin{cases} \delta_i\, \delta_{\hat{z}}\, \delta_\varphi & \text{if } \; i = j\,, \quad \text{'click'} \\ \delta_i\, \delta_{\vec{n}}\, \delta_\varnothing & \text{if } \; i \neq j\,. \quad \text{'no click'} \end{cases}$$

$$(i, \vec{n}, \varnothing) \xrightarrow{D_j} \begin{cases} \delta_i\, \delta_{\hat{z}}\, \delta_\varnothing & \text{if } \; i = j\,, \quad \text{'click'} \\ \delta_i\, \delta_{\vec{n}}\, \delta_\varnothing & \text{if } \; i \neq j\,. \quad \text{'no click'} \end{cases}$$

Local deterministic *gate !!*





REAL $(i = j)$ GHOST / EMPTY $(i \neq j)$

Beam splitter

Action of **beam splitter** $B(\vartheta)$:

*The gate takes **both** particles (REAL & GHOST) and depending on their inner states $\vec{n} = (\theta, \phi)$ and φ produces probabilistic **mixture** of **two** situations:*

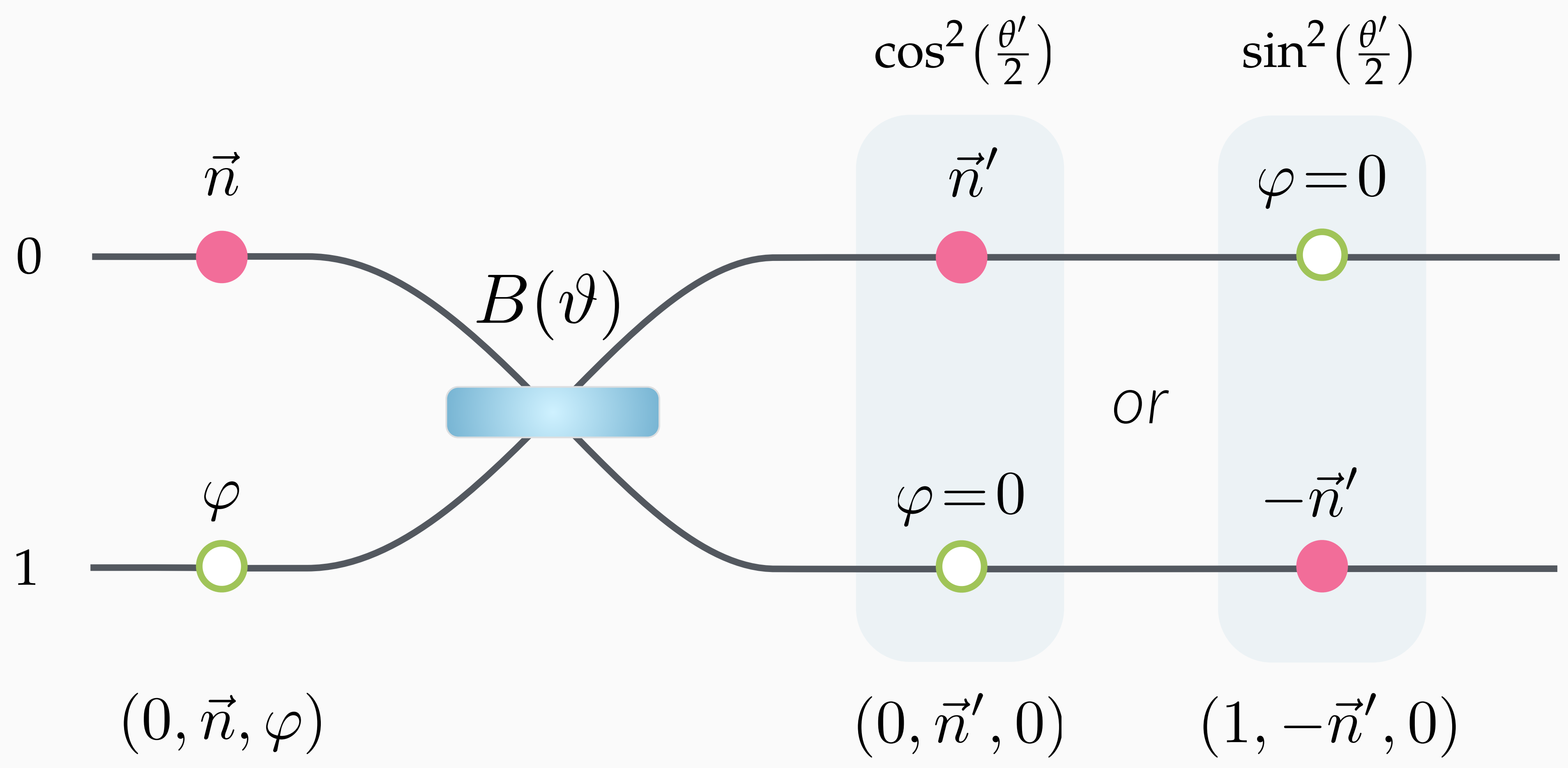
- *particles **remain** in their respective channels,*
- *particles are **swapped**,*

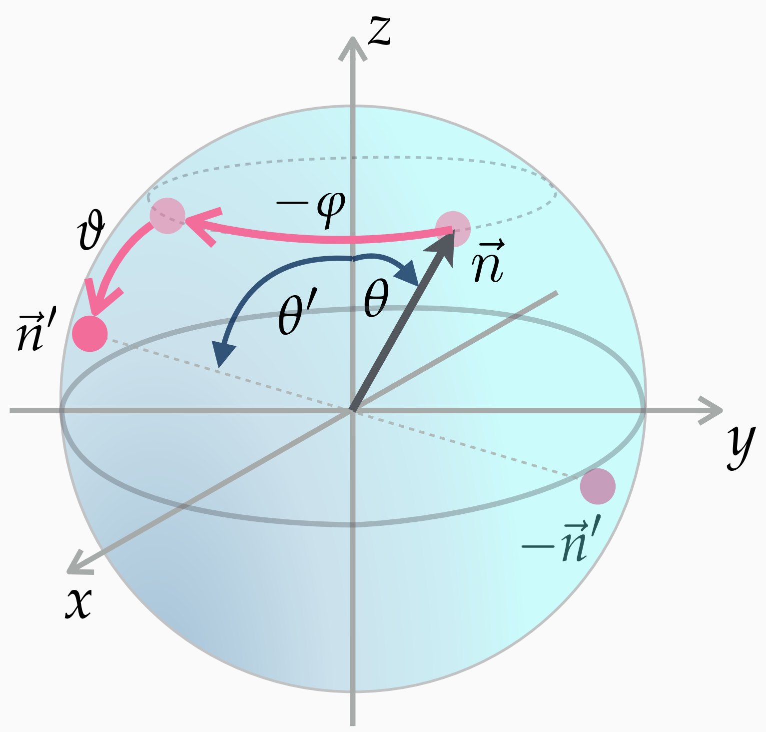
changing $\vec{n} \to \vec{n}'$ *and* $\varphi \to 0$.

$$(i, \vec{n}, \varphi) \xrightarrow{B(\vartheta)} \cos^2\left(\tfrac{\theta'}{2}\right) \delta_i\, \delta_{\vec{n}'}\, \delta_0 \;+\; \sin^2\left(\tfrac{\theta'}{2}\right) \delta_{\bar{i}}\, \delta_{-\vec{n}'}\, \delta_0$$

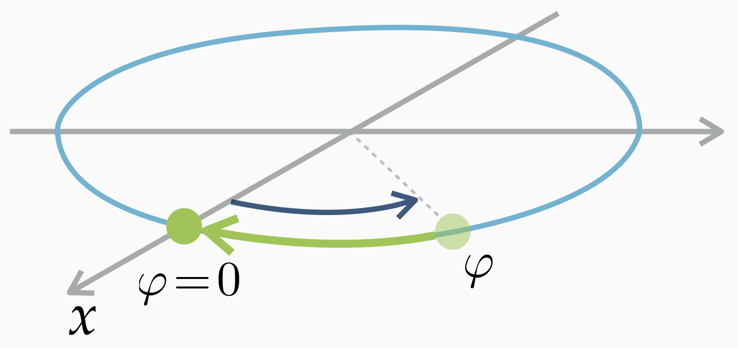
where: $\vec{n}' = (\theta', \phi') = R_x(\vartheta)\, R_z(-\varphi)\, \vec{n}$.

Local stochastic gate !!





REAL



GHOST

Beam splitter

Action of **beam splitter** $B(\vartheta)$:

The gate sets $\vec{n} \to \hat{z}$ for the REAL particle

creates a GHOST in the EMPTY channel $\varnothing \to \varphi = 0$

and acts accordingly, i.e.:

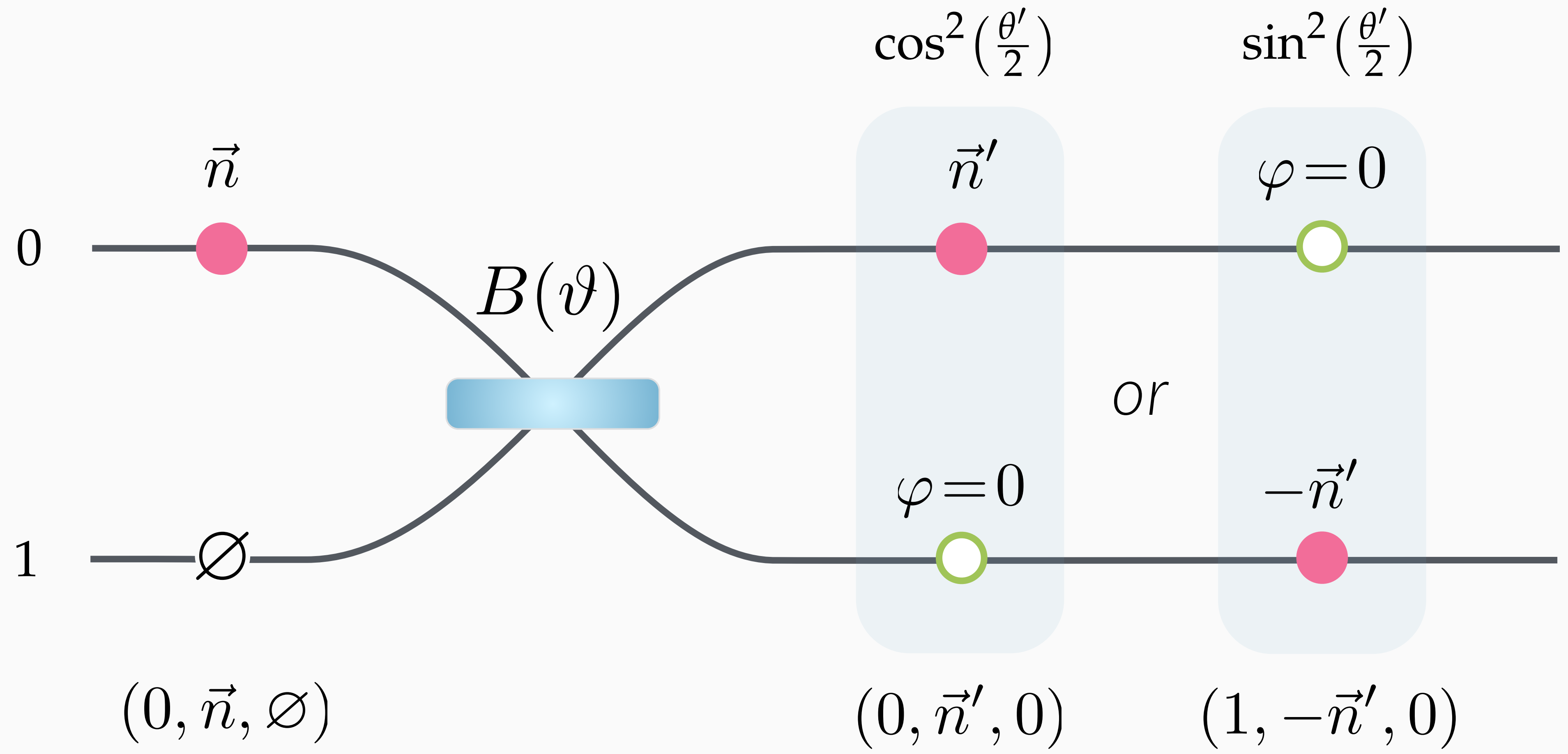
- particles **remain** in their respective channels,
- particles are **swapped**,

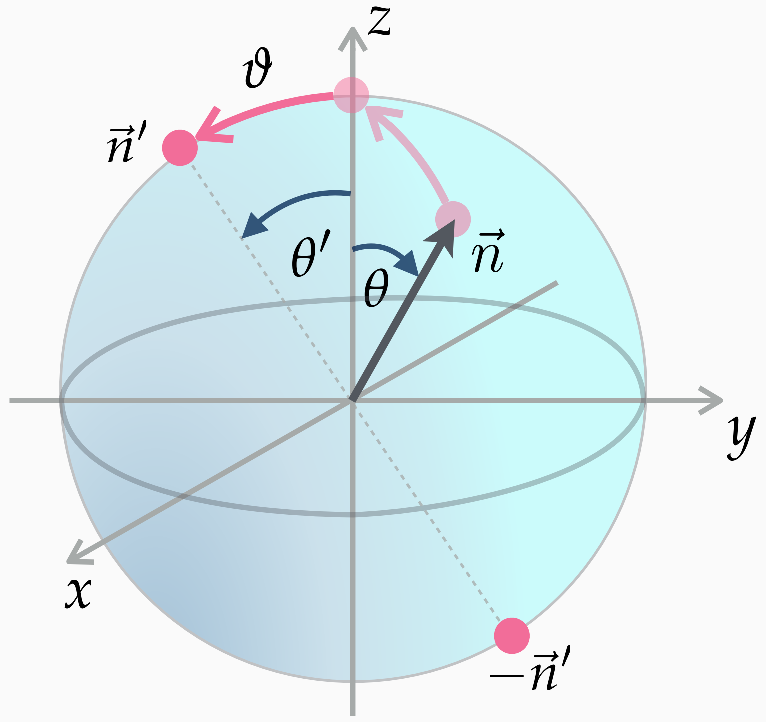
changing $\vec{n} \to \vec{n}'$ and $\varphi \to 0$.

$$(i, \vec{n}, \varnothing) \xrightarrow{B(\vartheta)} \cos^2\left(\tfrac{\theta'}{2}\right) \delta_i\, \delta_{\vec{n}'}\, \delta_0 \;+\; \sin^2\left(\tfrac{\theta'}{2}\right) \delta_{\bar{i}}\, \delta_{-\vec{n}'}\, \delta_0$$

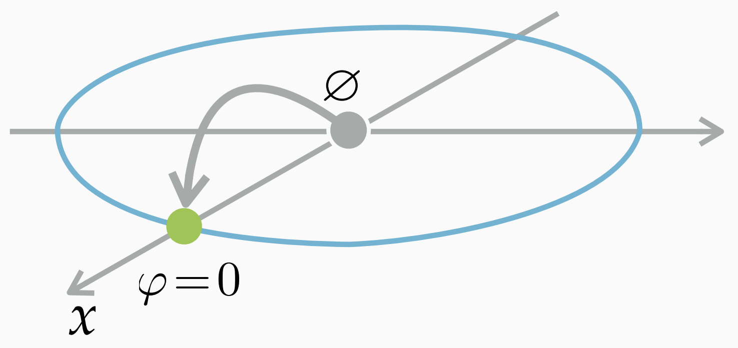
where: $\vec{n}' = (\theta', \phi') = \left(\vartheta, \pm\tfrac{\pi}{2}\right) = R_x(\vartheta)\, \hat{z}\,.$

Local stochastic gate !!





REAL



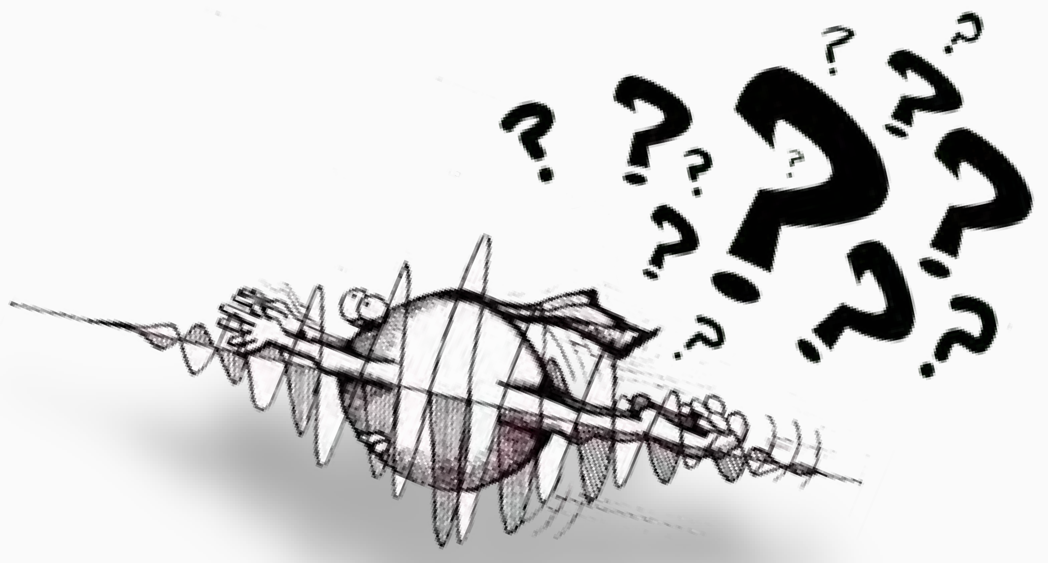
GHOST / EMPTY

Building the model

Plan of action

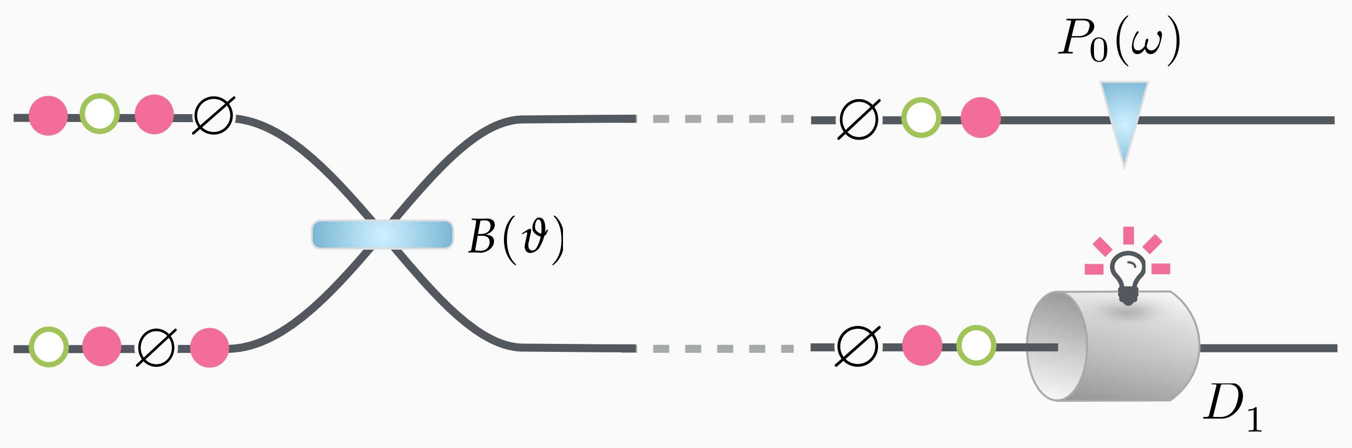
*Indeed, the stochastic model '**resembles**' interferometric circuits (locality !!).*

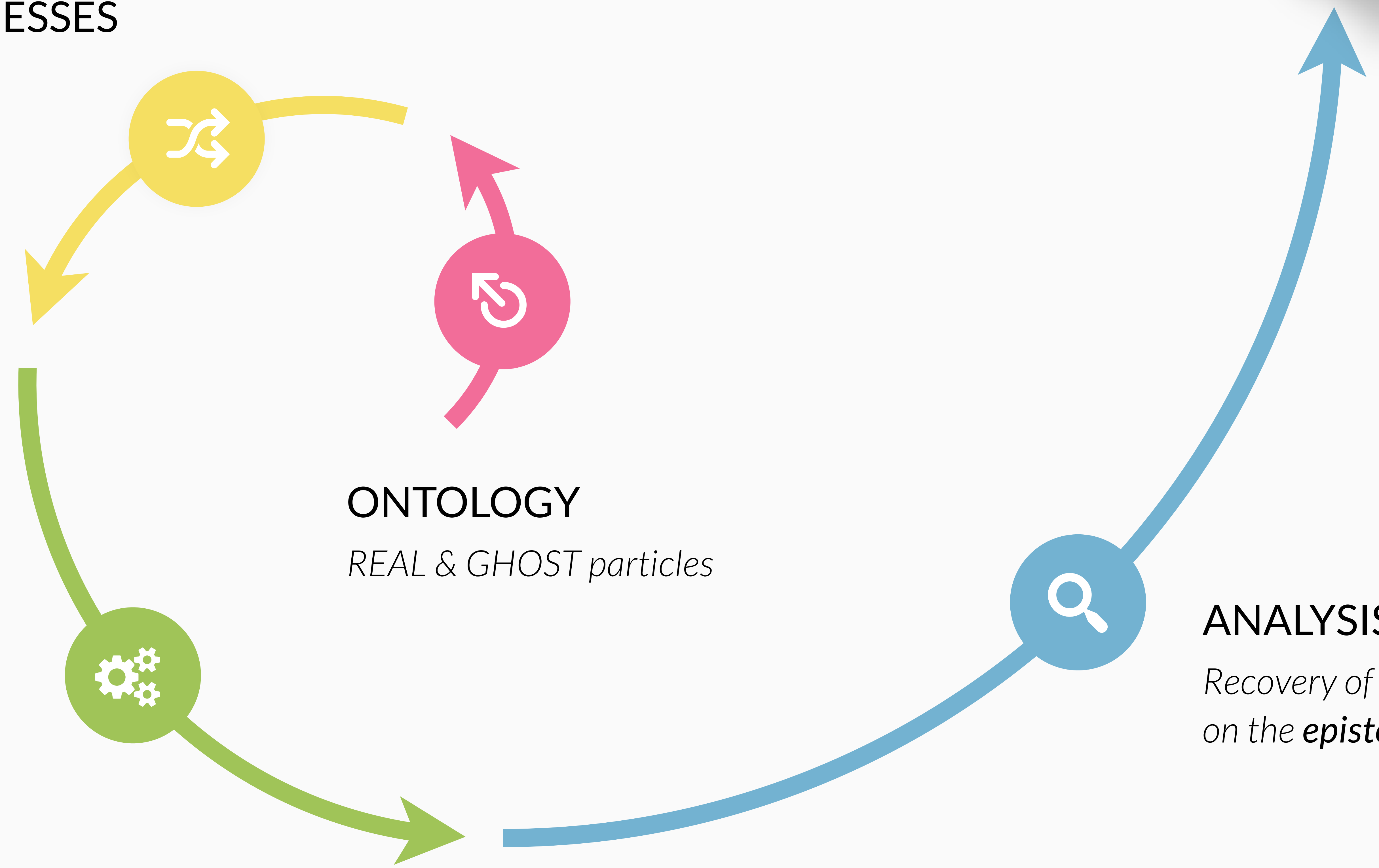
*How does it compare with **quantum predictions**? Where is the **wave function**?*



STOCHASTIC PROCESSES

Local vs. non-local gates





ONTOLOGY

REAL & GHOST particles

BASIC BUILDING BLOCKS

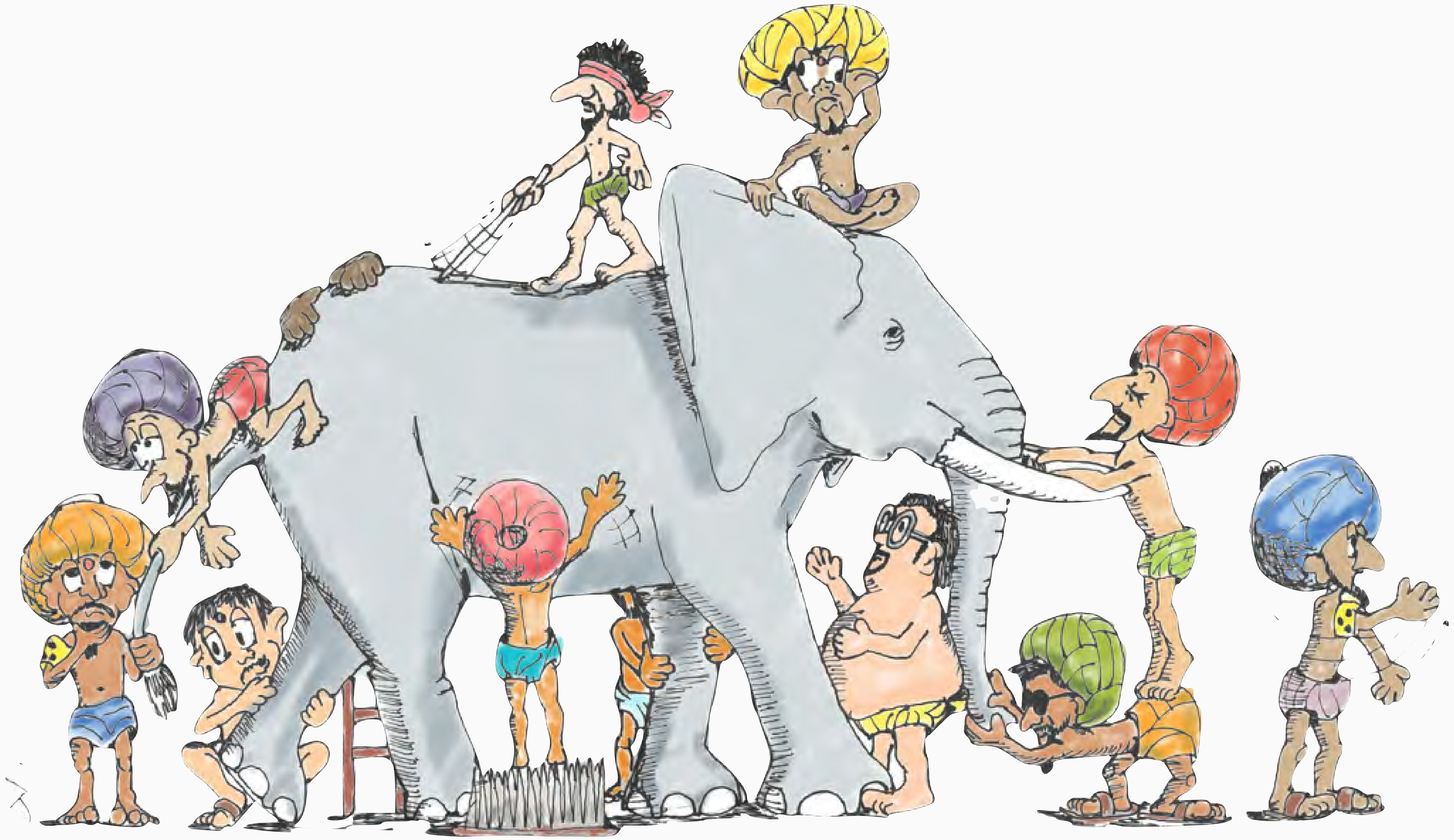
Limited set of stochastic gates

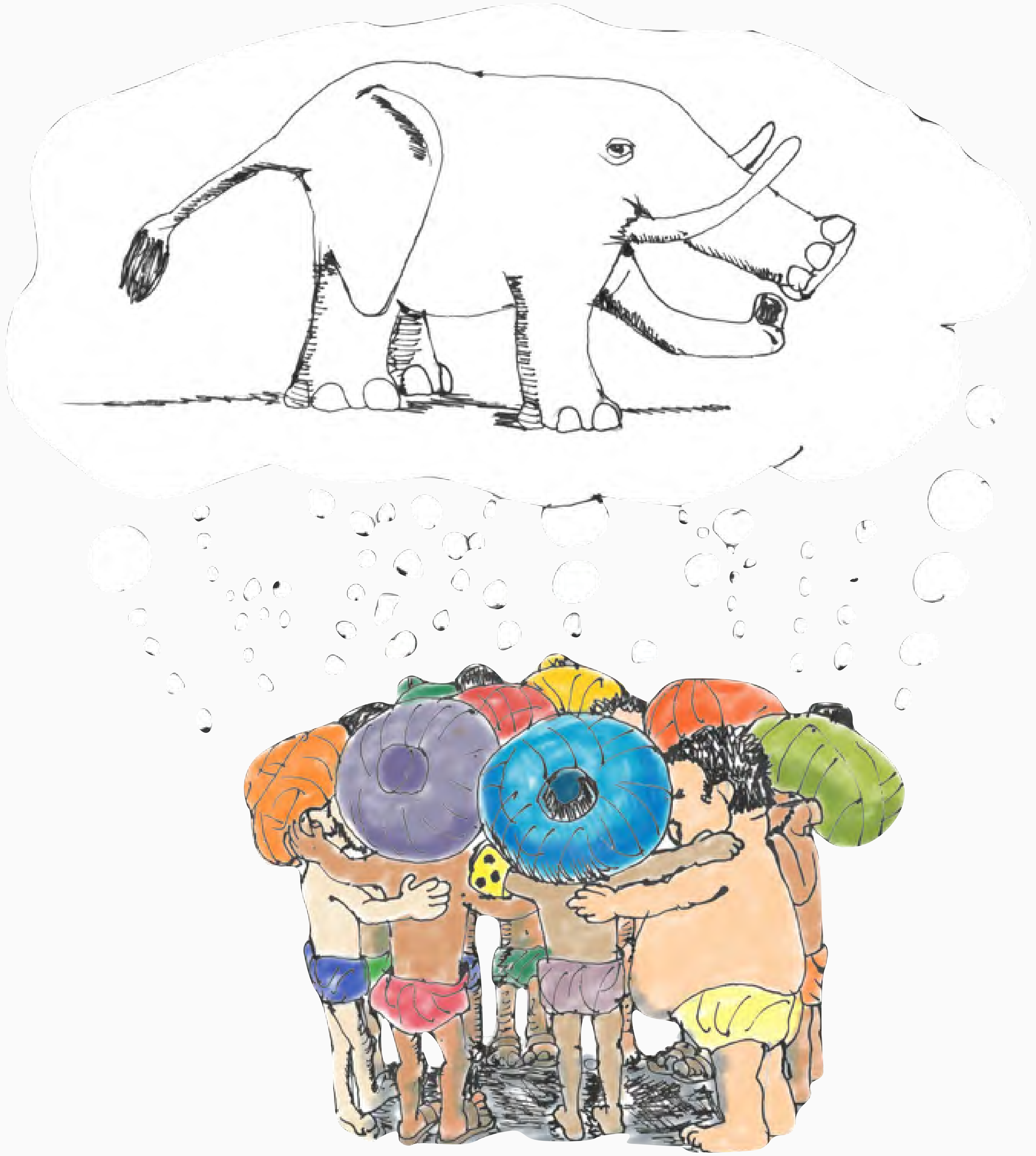
ANALYSIS OF THE MODEL

*Recovery of quantum predictions on the **epistemic level***

Ontic vs. Epistemic

Blind man and an elephant





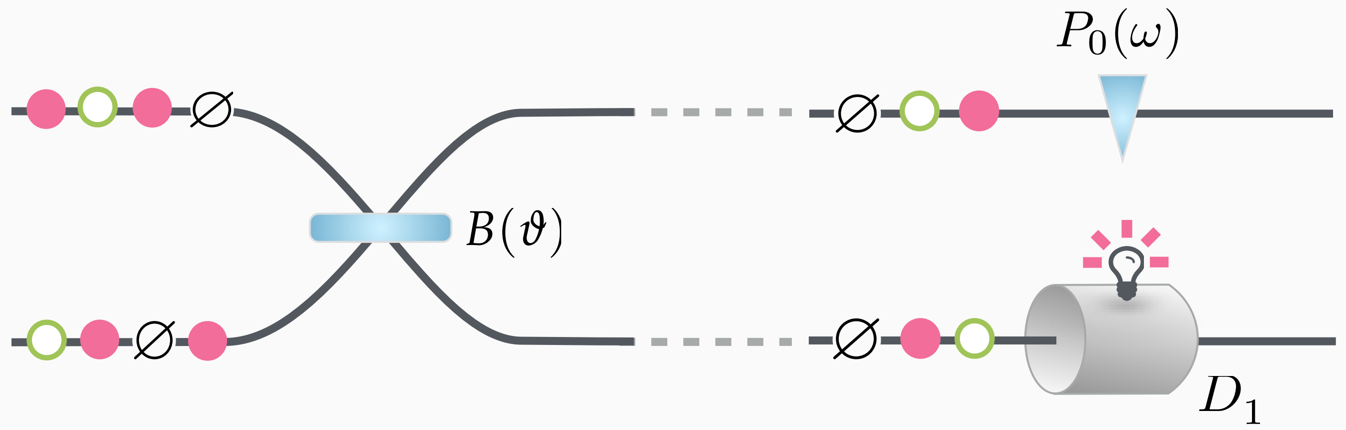
"We have to remember that what we observe is not nature in itself, but ***nature exposed to our method of questioning****."*

— Werner Heisenberg

Epistemic desideratum

Agent under constraints

Ontic perspective

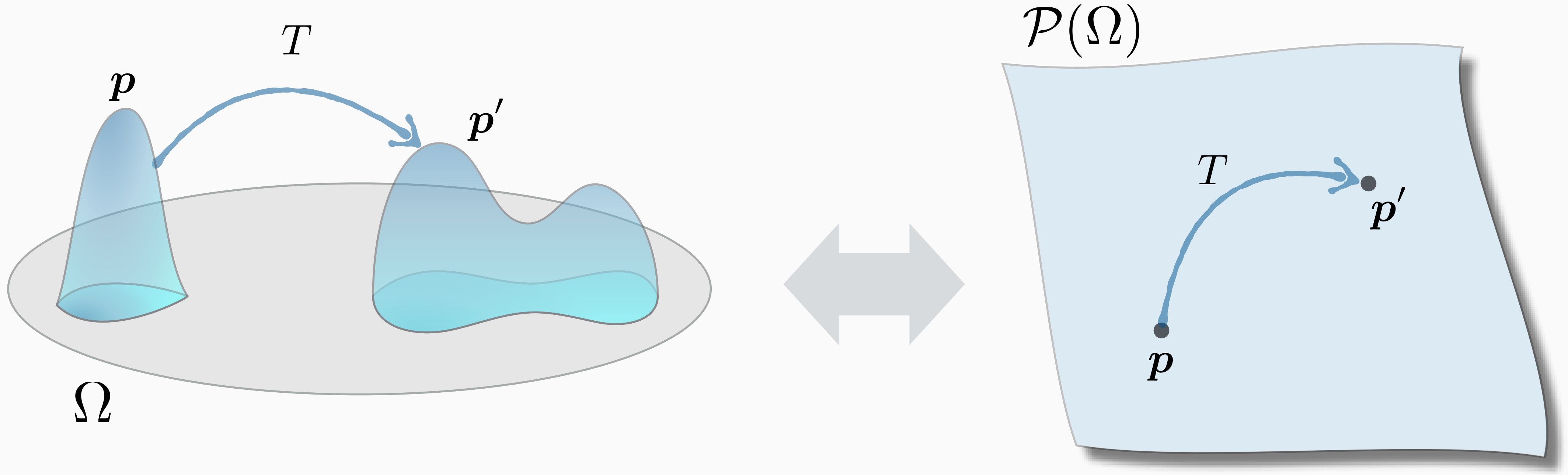


$$\Omega \equiv \{0,1\} \times S^2 \times S^{1^*} \ni (i, \vec{n}, \varphi) \;\text{ or }\; (i, \vec{n}, \varnothing)$$

where is REAL particle

inner state of REAL particle

inner state of GHOST particle or EMPTY



Epistemic perspective

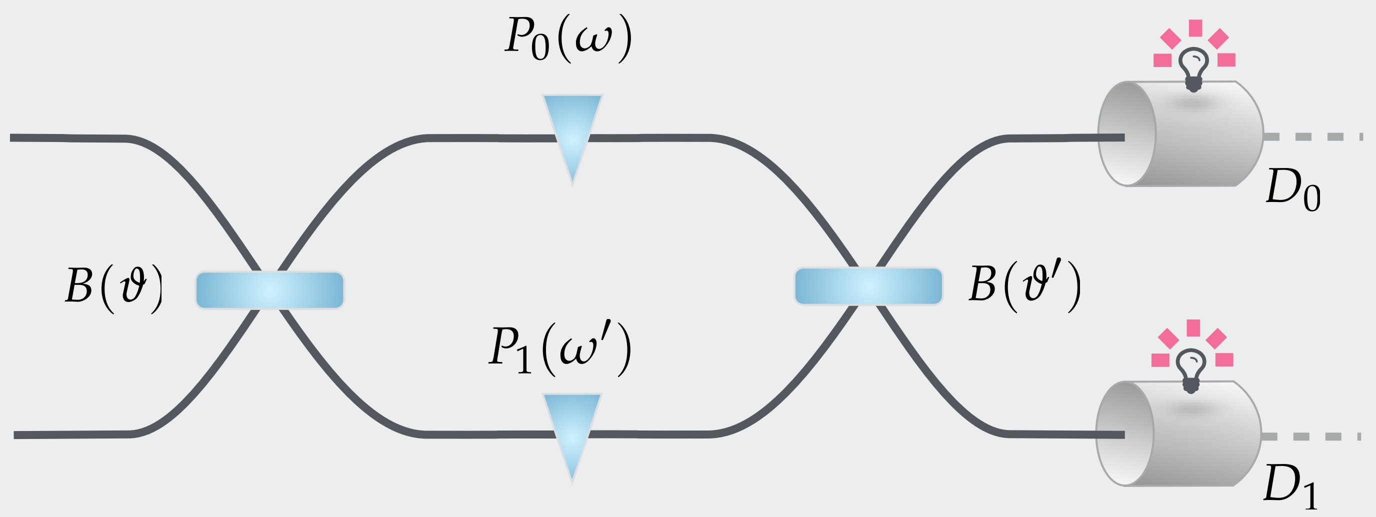
Available resources:

Phase shifters

Beam splitters

Detectors (post-selection)

Probabilistic mixing



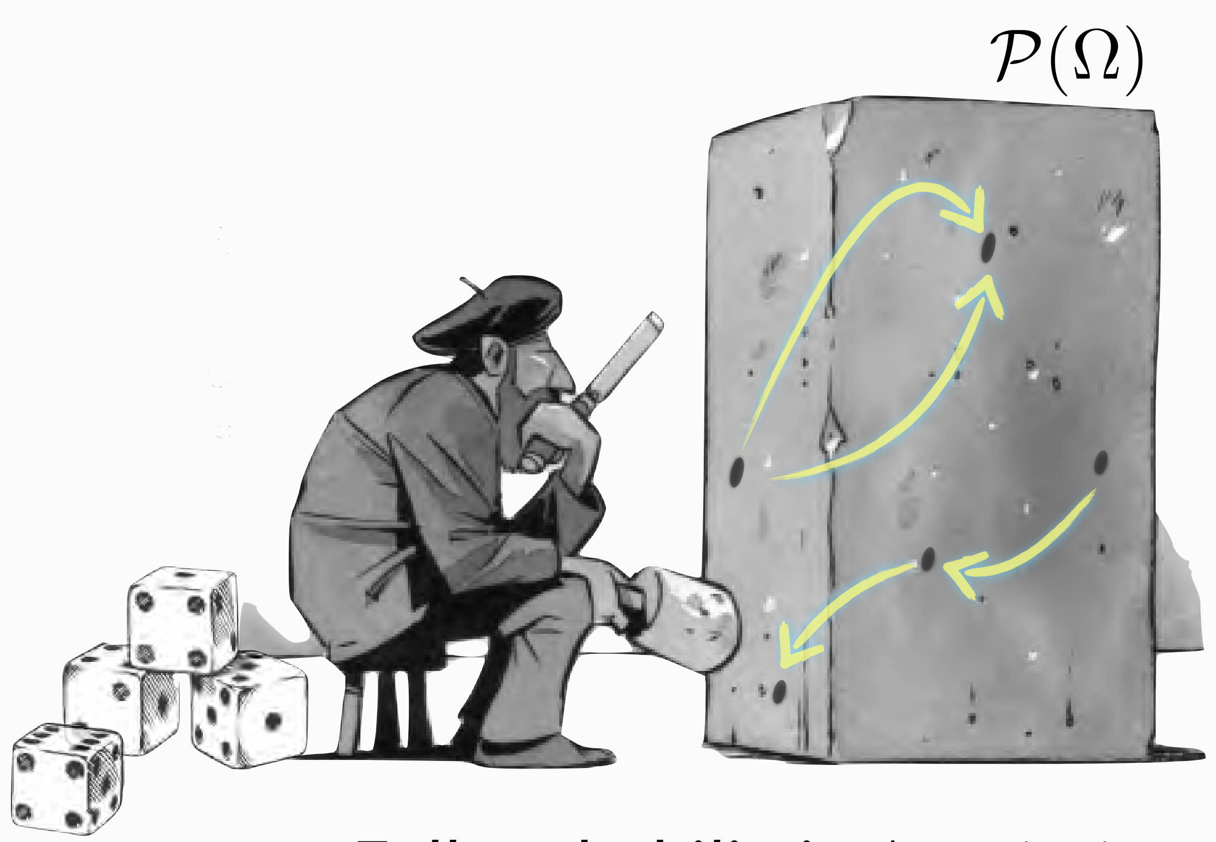
*The agent '**sees**' the model only through experiments i.e. using only a **limited choice of gates** .*

Epistemic desideratum
Agent under constraints

Operational description of the model

- ***Which distributions*** *in $\mathcal{P}(\Omega)$ can be prepared by the agent according to the rules of the model ?*
- ***How do they transform*** *and what information can be learned under the action of conceivable circuits ?*

- *What is the* ***minimal description*** *which is enough to predict behaviour of the system as '****seen****' by the agent ?*



Full probabilistic *description*

What is the geometry of ***accessible states***

Epistemic perspective

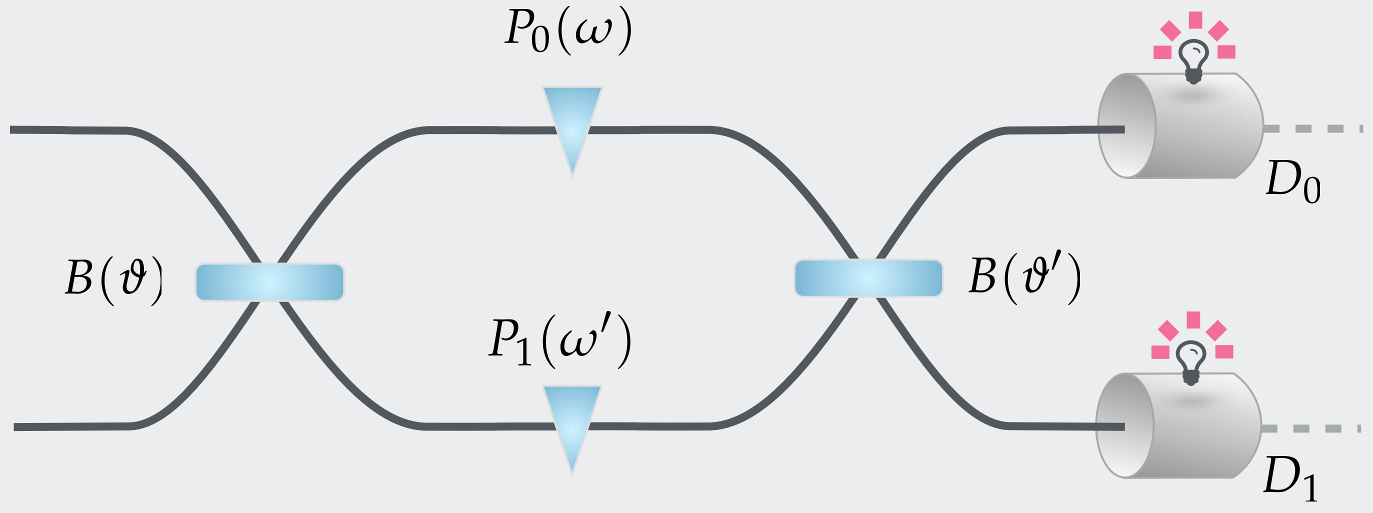
Available resources:

Phase shifters

Beam splitters

Detectors (post-selection)

Probabilistic mixing



*The agent '****sees****' the model only through experiments i.e. using only a* ***limited choice of gates*** *.*

Analysis of the Model

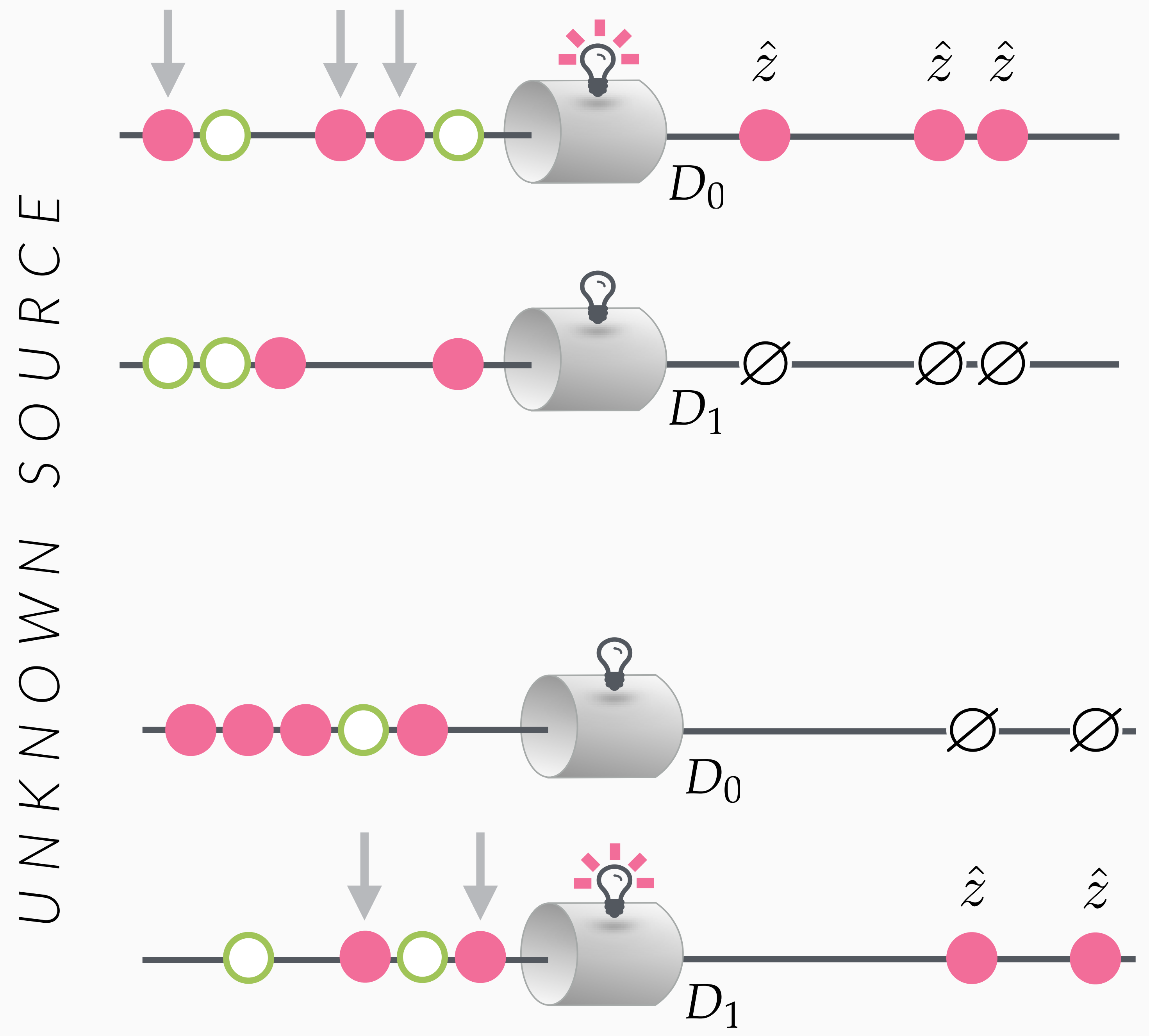
Initialisation

How to make a circuit work, i.e. **INITIALISE** ?

Make sure there is only **one** REAL particle in the circuit, with a GHOST / EMPTY in ***another*** channel.

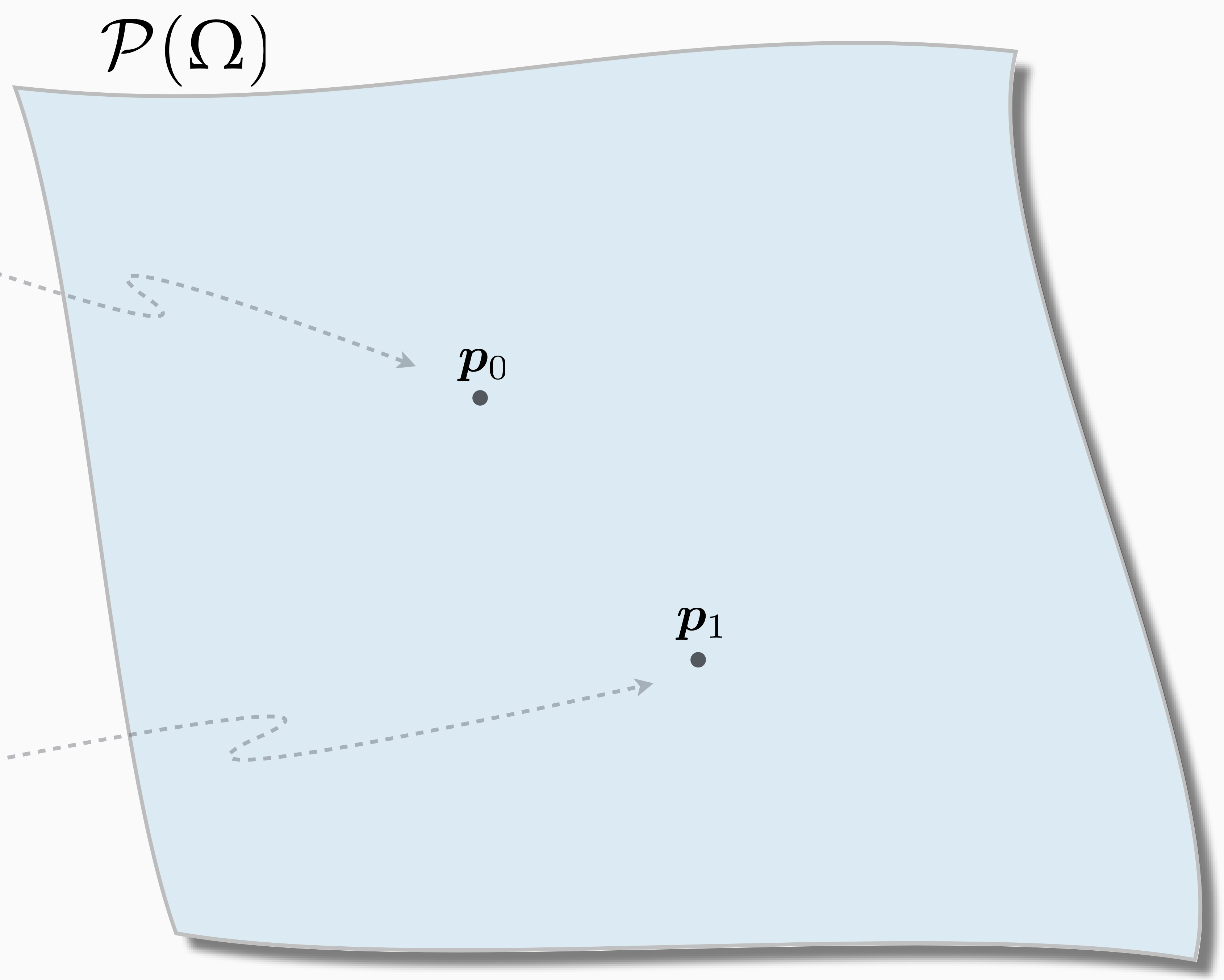
Key assumption:

Only **single** REAL particle present in the circuit, ***possibly*** accompanied by a GHOST in another channel.



$$\boldsymbol{p}_0 \equiv \delta_0 \, \delta_{\hat{z}} \, \delta_{\varnothing} \in \mathcal{P}(\Omega)$$

$$\boldsymbol{p}_1 \equiv \delta_1 \, \delta_{\hat{z}} \, \delta_{\varnothing} \in \mathcal{P}(\Omega)$$

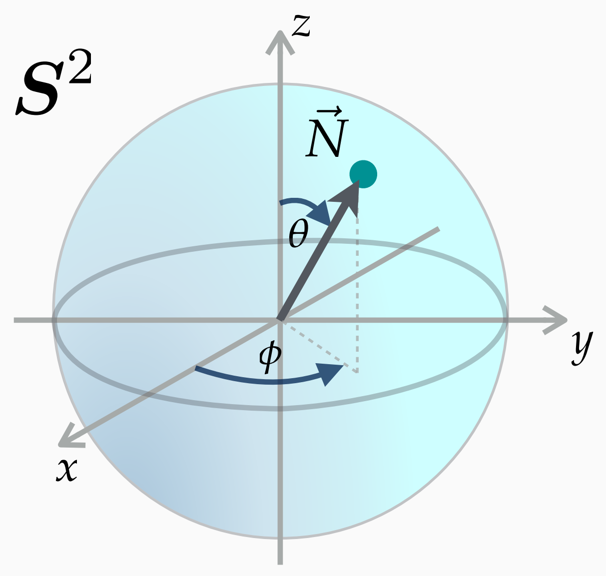


(*) One can use two detectors or detector and blocker.

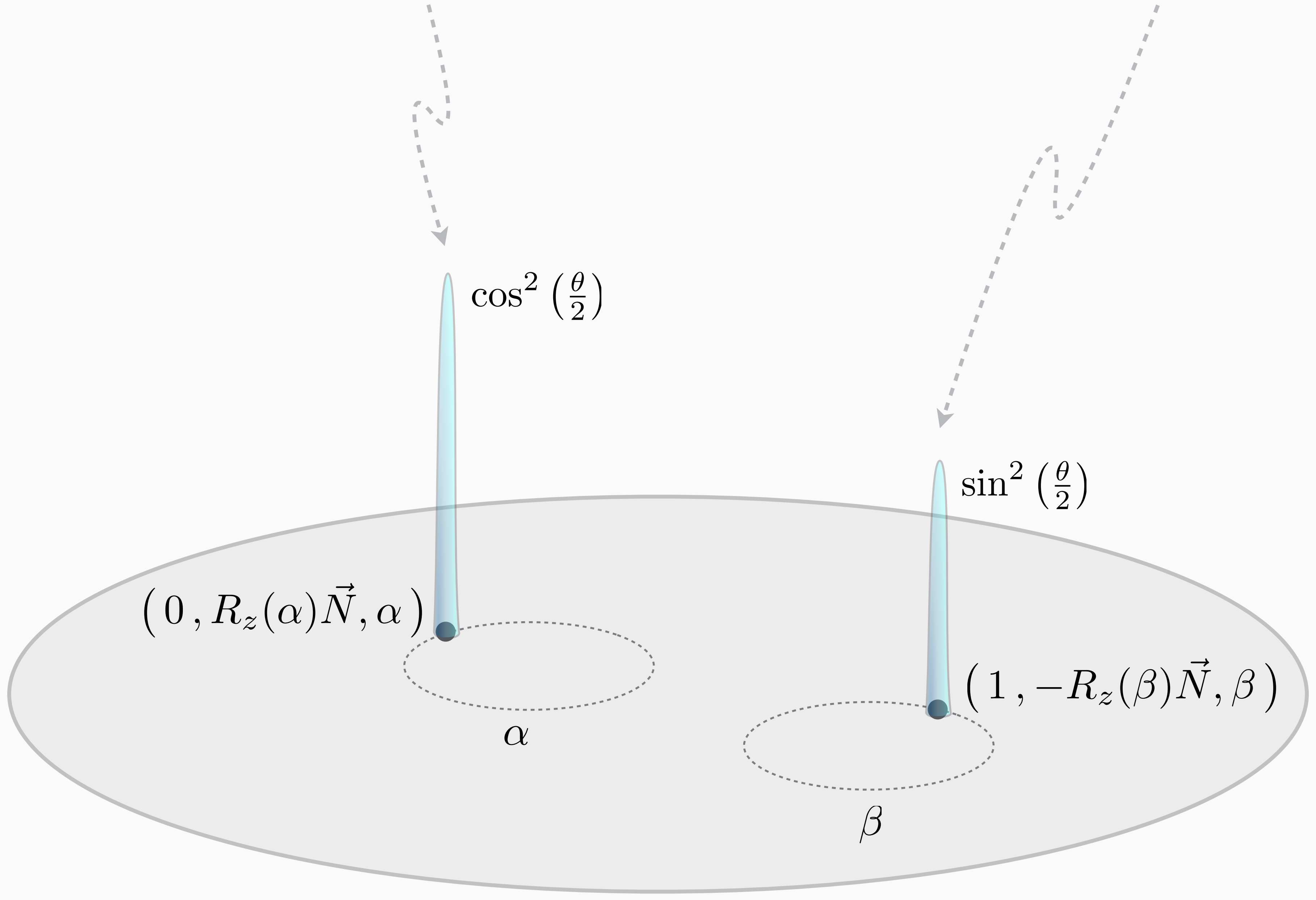
Definition.

For each $\vec{N} = (\theta, \phi) \in \boldsymbol{S}^2$, *we define a class of distributions:*

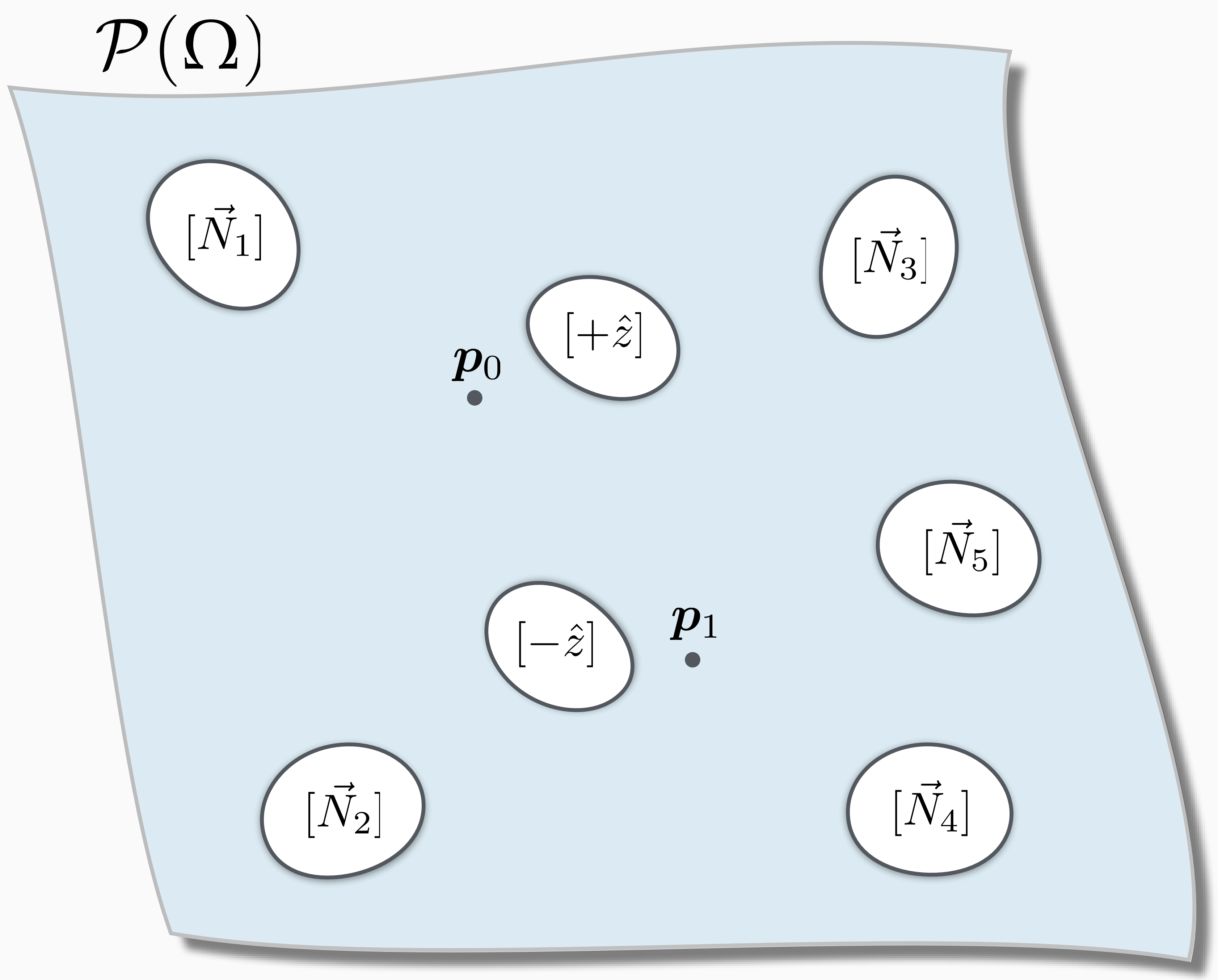
$$[\vec{N}] \equiv \left\{ \cos^2\left(\frac{\theta}{2}\right) \delta_0\, \delta_{R_z(\alpha)\,\vec{N}}\, \delta_\alpha + \sin^2\left(\frac{\theta}{2}\right) \delta_1\, \delta_{-R_z(\beta)\,\vec{N}}\, \delta_\beta \;:\; \alpha, \beta \in [0, 2\pi) \right\} \subset \mathcal{P}(\Omega)$$



Labelling of classes $[\vec{N}]$



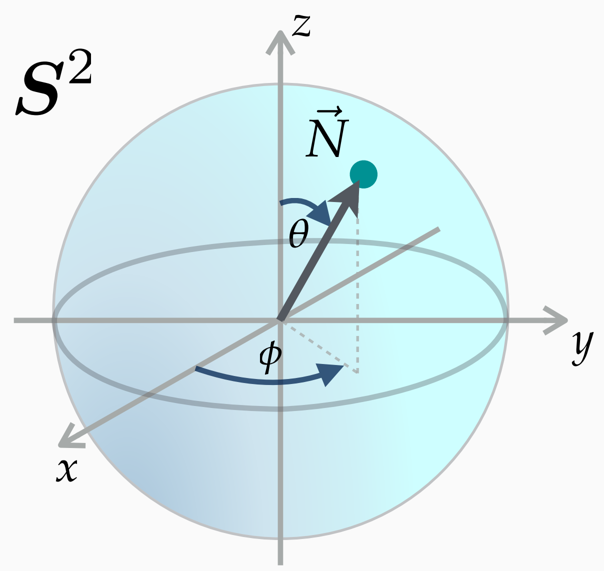
$$\Omega \equiv \{0,1\} \times S^2 \times S^{1*}$$



Definition.

For each $\vec{N} = (\theta,\phi) \in \boldsymbol{S}^2$, *we define a class of distributions:*

$$[\vec{\mathrm{N}}] \equiv \left\{ \cos^2\left(\tfrac{\theta}{2}\right)\, \delta_0\, \delta_{R_z(\alpha)\,\vec{\mathrm{N}}}\, \delta_\alpha \;+\; \sin^2\left(\tfrac{\theta}{2}\right)\, \delta_1\, \delta_{-R_z(\beta)\,\vec{\mathrm{N}}}\, \delta_\beta \;:\; \alpha,\beta \in [0,2\pi) \right\} \subset \mathcal{P}(\Omega)$$

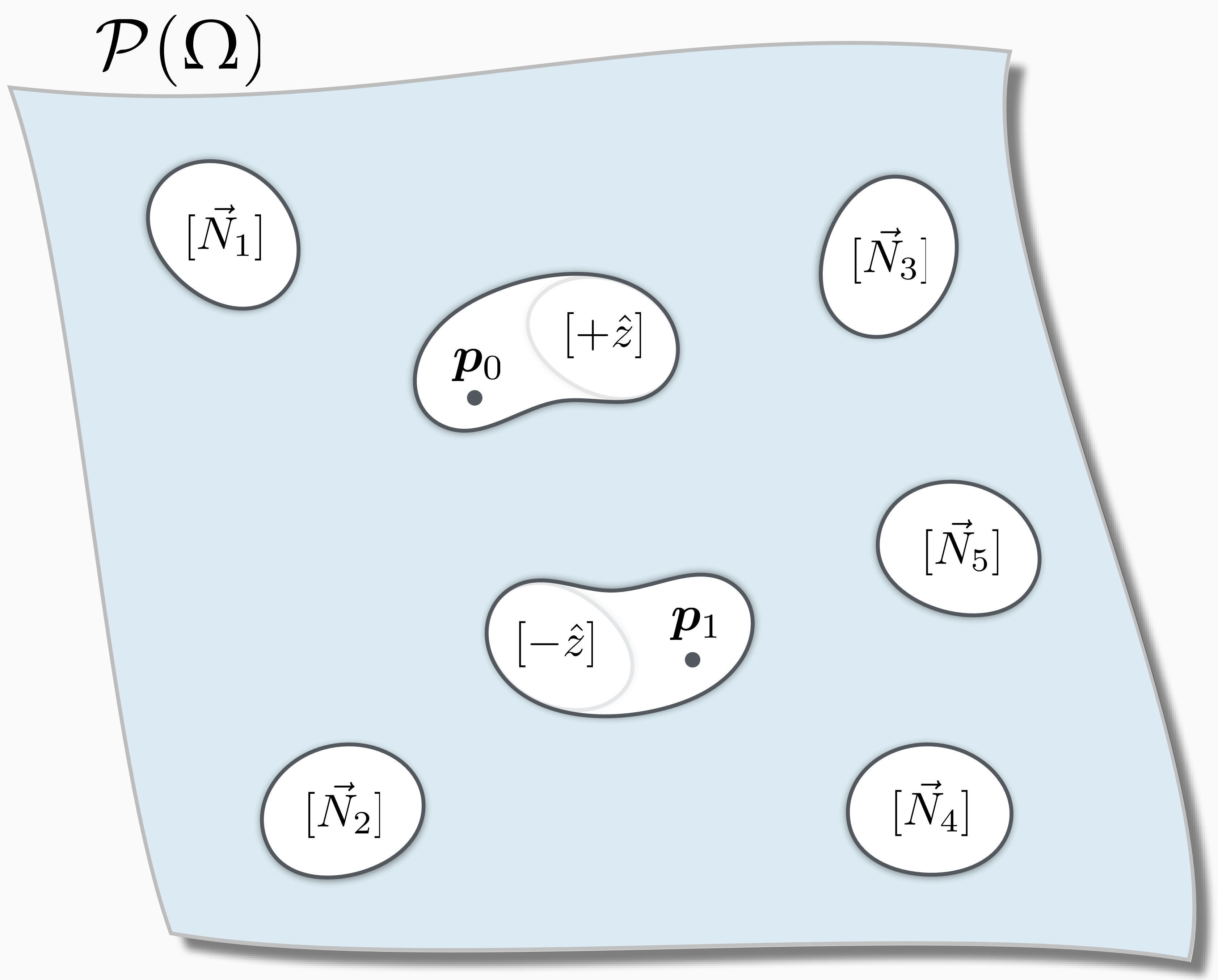


Labelling of classes $[\vec{N}]$

In particular:

$$[+\hat{z}] \equiv \left\{ \delta_0\, \delta_{\hat{z}}\, \delta_\alpha \;:\; \alpha \in [0,2\pi) \right\}$$

$$[-\hat{z}] \equiv \left\{ \delta_1\, \delta_{\hat{z}}\, \delta_\beta \;:\; \beta \in [0,2\pi) \right\}$$



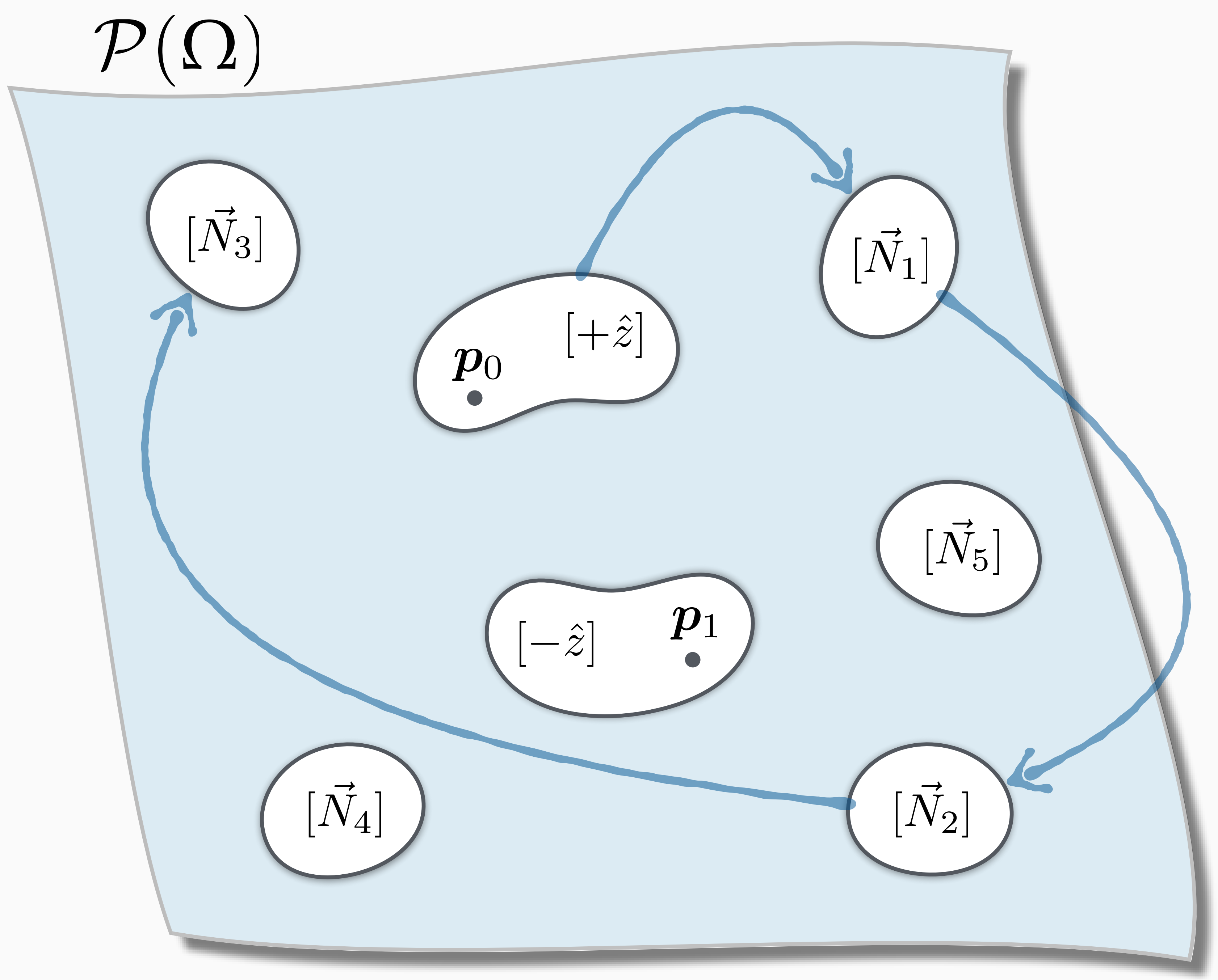
For $\vec{N} = \pm\hat{z}$, *we augment* $[\pm\hat{z}]$ *to account for the EMPTY path:*

$$[+\hat{z}] \longrightarrow [+\hat{z}] \cup \left\{ \delta_0\, \delta_{\vec{n}}\, \delta_\varnothing : \vec{n} \in S^2\,,\ \vec{n} \neq -\hat{z} \right\}$$

$$[-\hat{z}] \longrightarrow [-\hat{z}] \cup \left\{ \delta_1\, \delta_{\vec{n}}\, \delta_\varnothing : \vec{n} \in S^2\,,\ \vec{n} \neq -\hat{z} \right\}$$

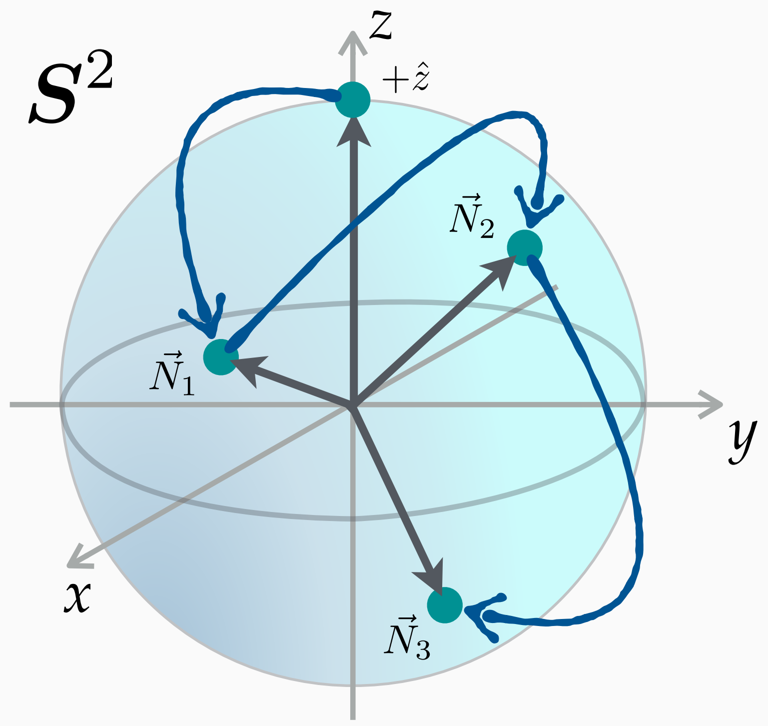
Lemma 1.

Phase shifters $P_i(\omega)$ *and beam splitters* $B(\vartheta)$ **do not** *leave outside the set* $\mathcal{E} \equiv \bigcup_{\vec{N}\in \boldsymbol{S}^2} [\vec{N}] \subsetneq \mathcal{P}(\Omega)$ *and classes map in a* **congruent** *manner, i.e.* $[\vec{\mathrm{N}}] \ni \boldsymbol{p} \xrightarrow{T} \boldsymbol{p}' \in [\vec{\mathrm{N}}_T]$.



For any sequence of
phase shifters *and* ***beam splitters***

Equivalent



Mapping of classes

Lemma 1.

*Phase shifters $P_i(\omega)$ and beam splitters $B(\vartheta)$ **do not** leave outside the set $\mathcal{E} \equiv \bigcup_{\vec{N}\in \boldsymbol{S}^2} [\vec{N}] \subsetneq \mathcal{P}(\Omega)$ and classes map in a **congruent** manner, i.e. $[\vec{N}] \ni \boldsymbol{p} \xrightarrow{T} \boldsymbol{p}' \in [\vec{N}_T]$.*

More specifically:

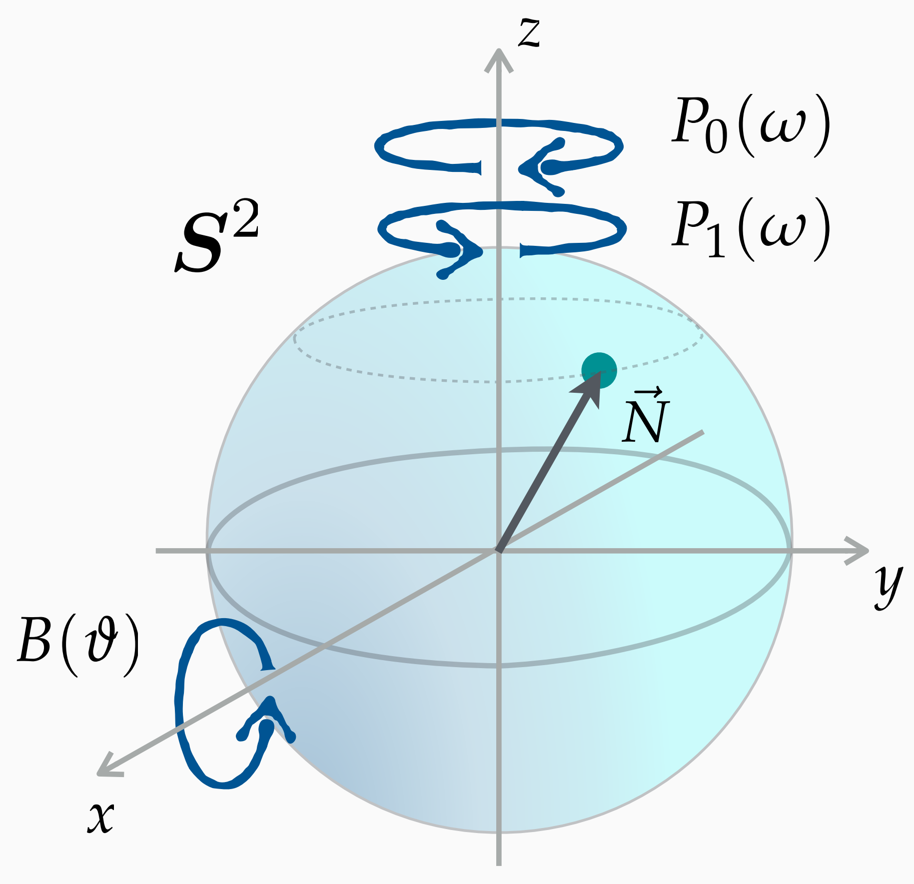
Phase shifters

$$[\vec{N}] \xrightarrow{P_0(\omega)} [R_z(-\omega)\,\vec{N}]$$

$$[\vec{N}] \xrightarrow{P_1(\omega)} [R_z(\omega)\,\vec{N}]$$

Beam splitters

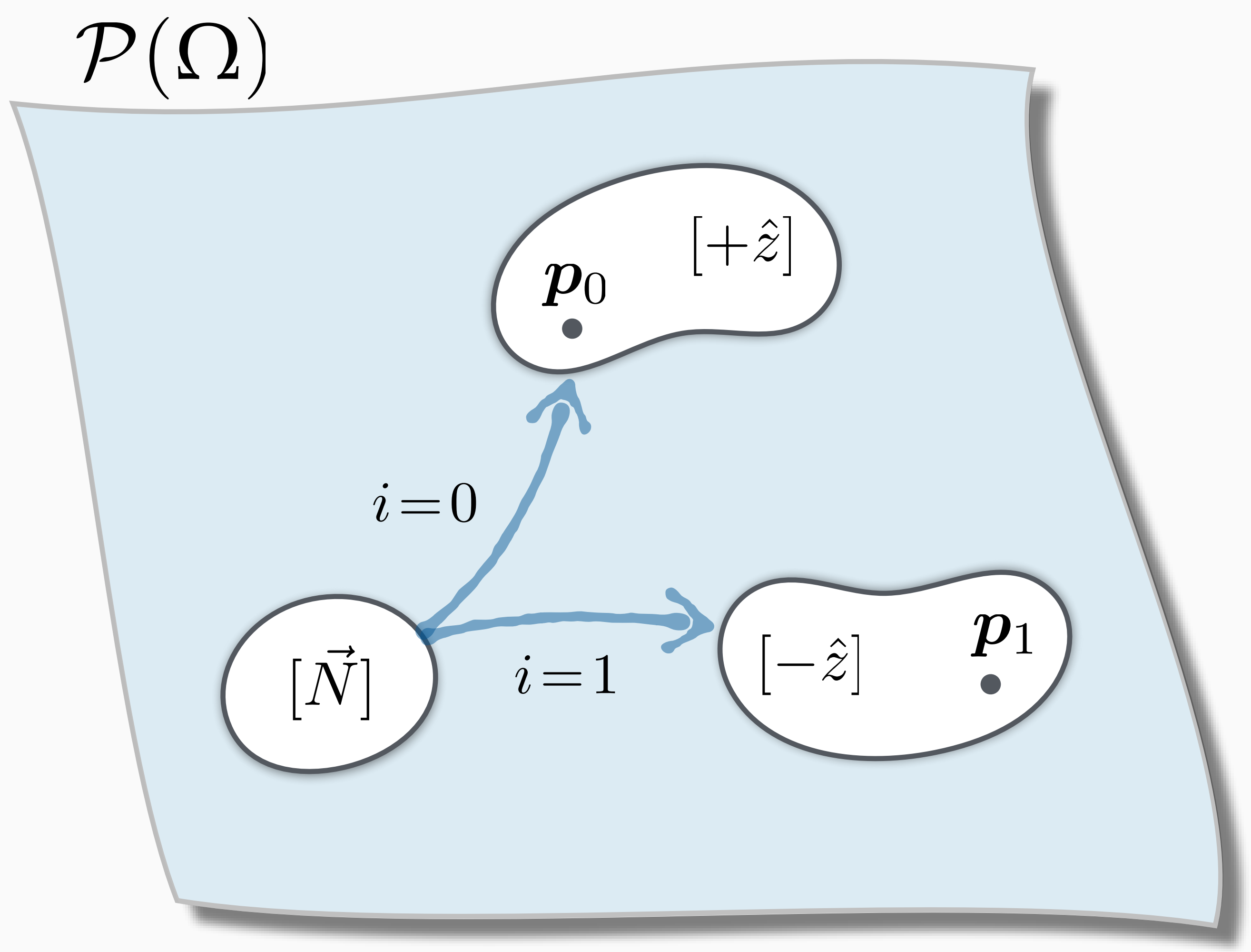
$$[\vec{N}] \xrightarrow{B(\vartheta)} [R_x(\vartheta)\,\vec{N}]$$



Mapping of classes

Lemma 2.

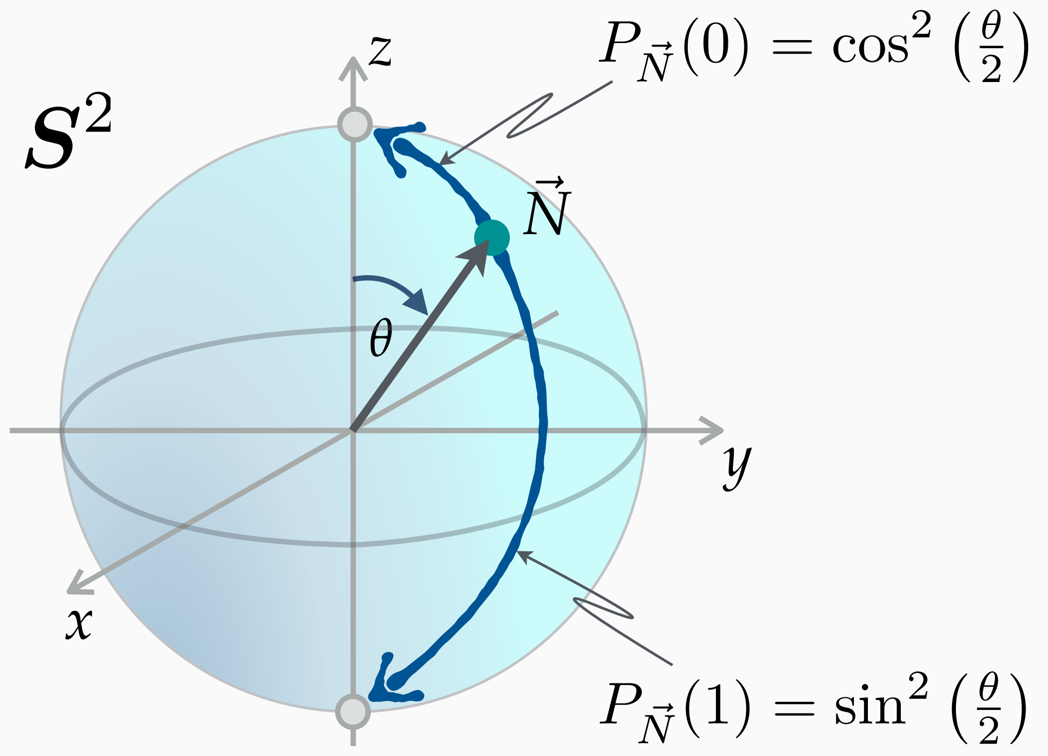
Detectors D_j reveal position of the REAL *particle $i = 0, 1$ (by a 'CLICK' / 'NO CLICK') and depending on the outcome yield a state in the respective class $[+\hat{z}]$ or $[-\hat{z}]$.*



More specifically:

Detectors

$$[\vec{N}] \xrightarrow{\quad D_j \quad} \begin{cases} [+\hat{z}] & \text{for outcome } i = 0 \text{ with } P_{\vec{N}}(0) = \cos^2\left(\frac{\theta}{2}\right) \\ [-\hat{z}] & \text{for outcome } i = 1 \text{ with } P_{\vec{N}}(1) = \sin^2\left(\frac{\theta}{2}\right) \end{cases}$$

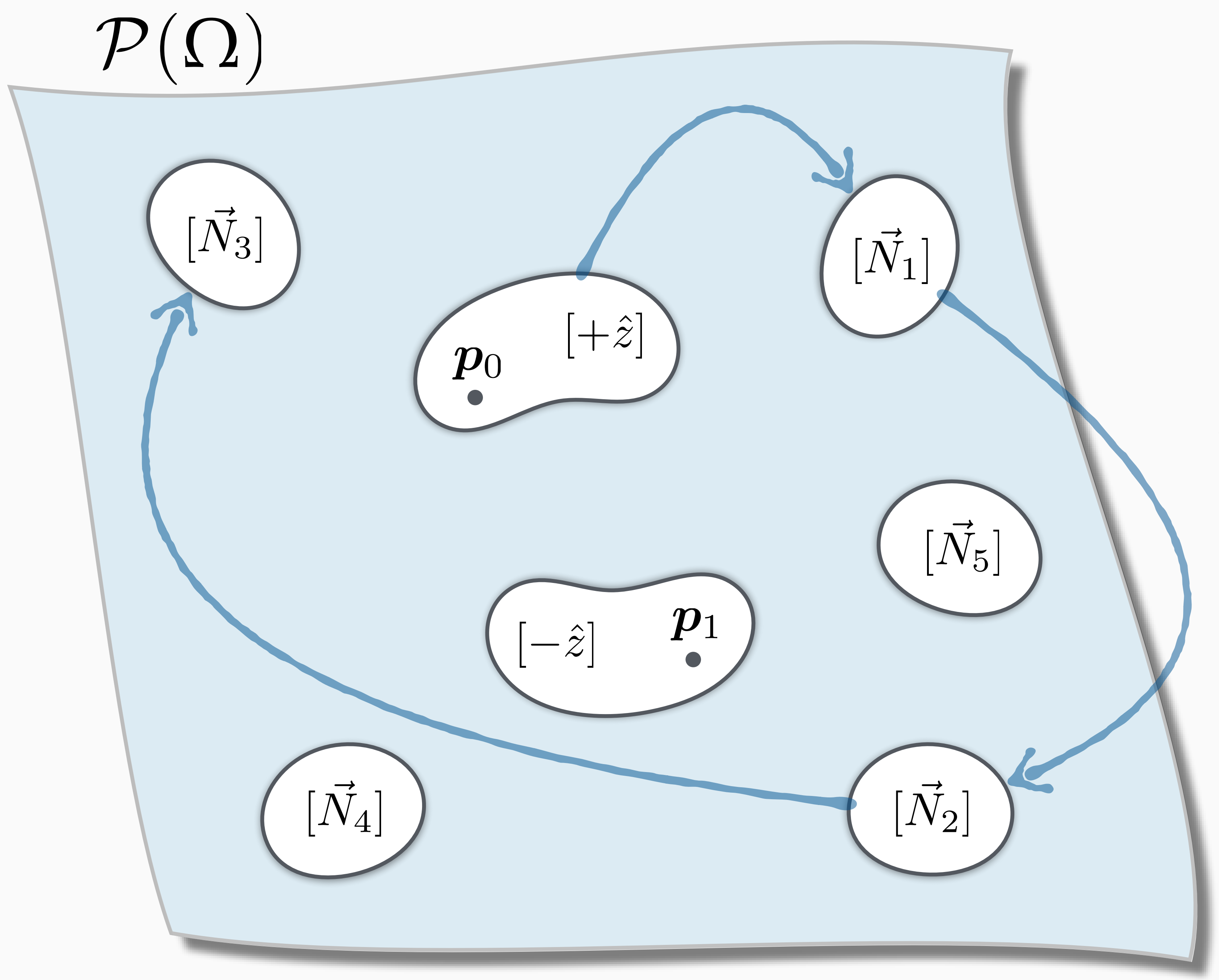


Mapping of classes

Analysis of the Model

Accessible states & Bloch sphere

Phase shifters**, **beam splitters
*& **detectors** with **post-selection***



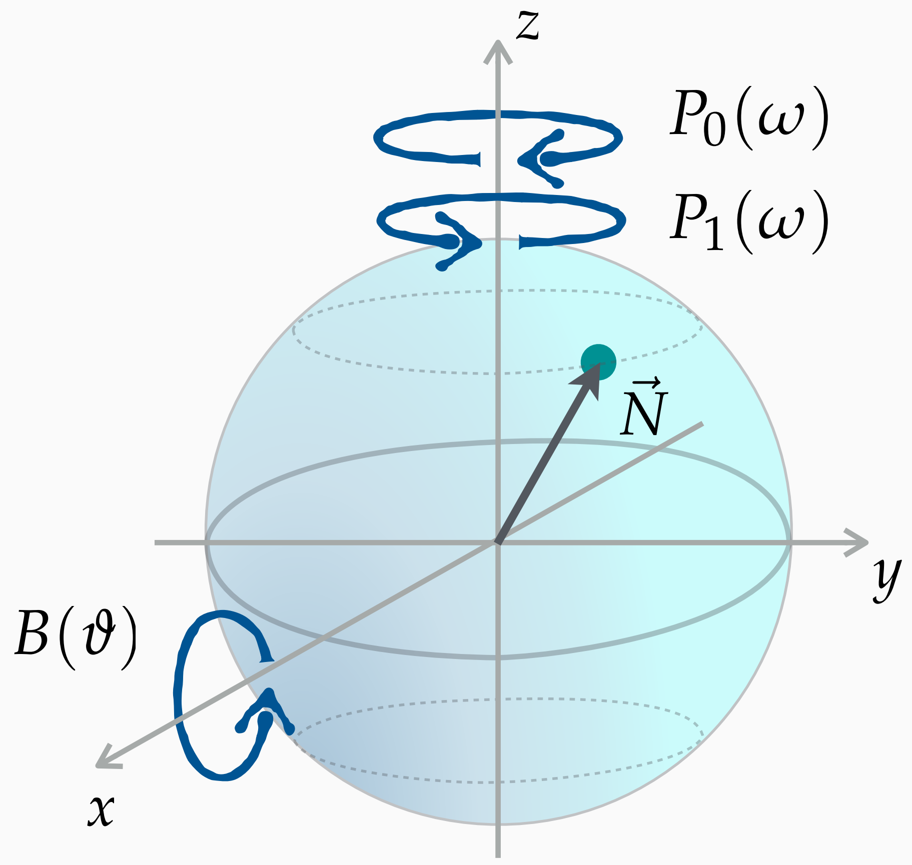
Equivalent

*Range of **accessible states**:*

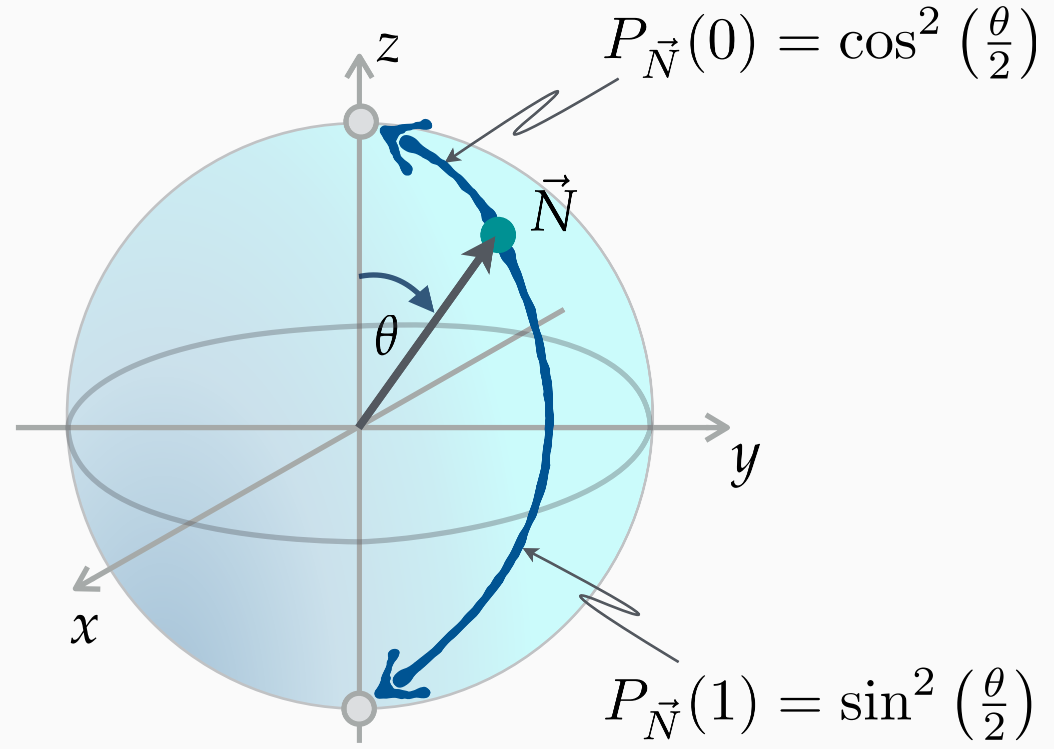
$$\mathcal{E} \;\equiv\; \bigcup_{\vec{N}\in\boldsymbol{S}^2} [\vec{N}] \;\subsetneq \mathcal{P}(\Omega)$$

***Indistinguishability** of distributions in $[\vec{N}]$*

***Transformation** rules on $\boldsymbol{S}^2$:*



Phase shifters** & **Beam splitters



Detectors** with **post-selection

*Equivalent to **Bloch sphere** representation !!!*

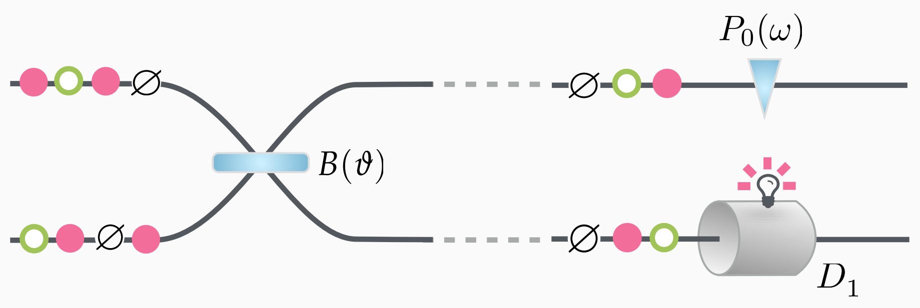
Summary

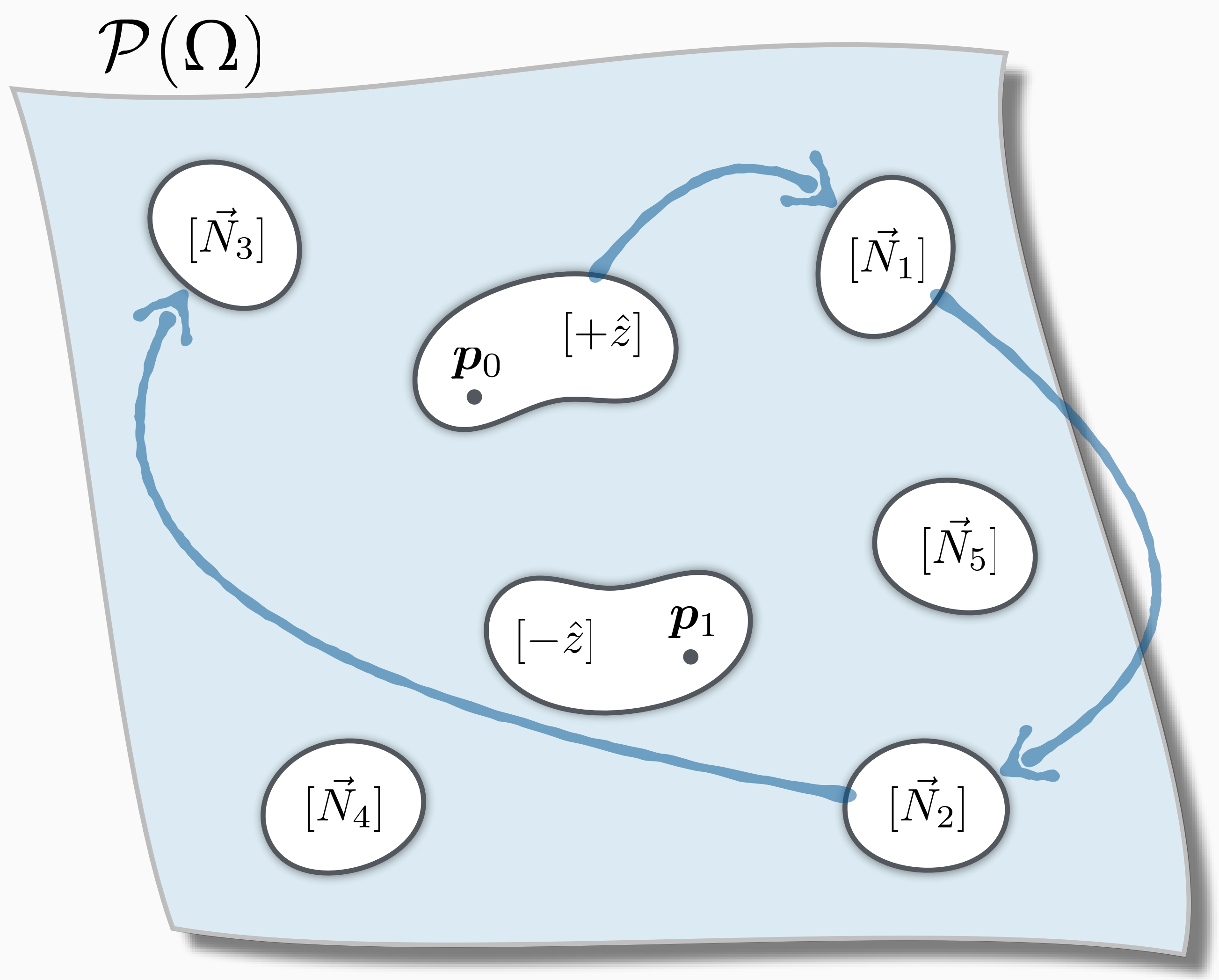
Agent under epistemic constraints

*REAL & GHOST particle ontology + **limited** set of **stochastic gates**.*

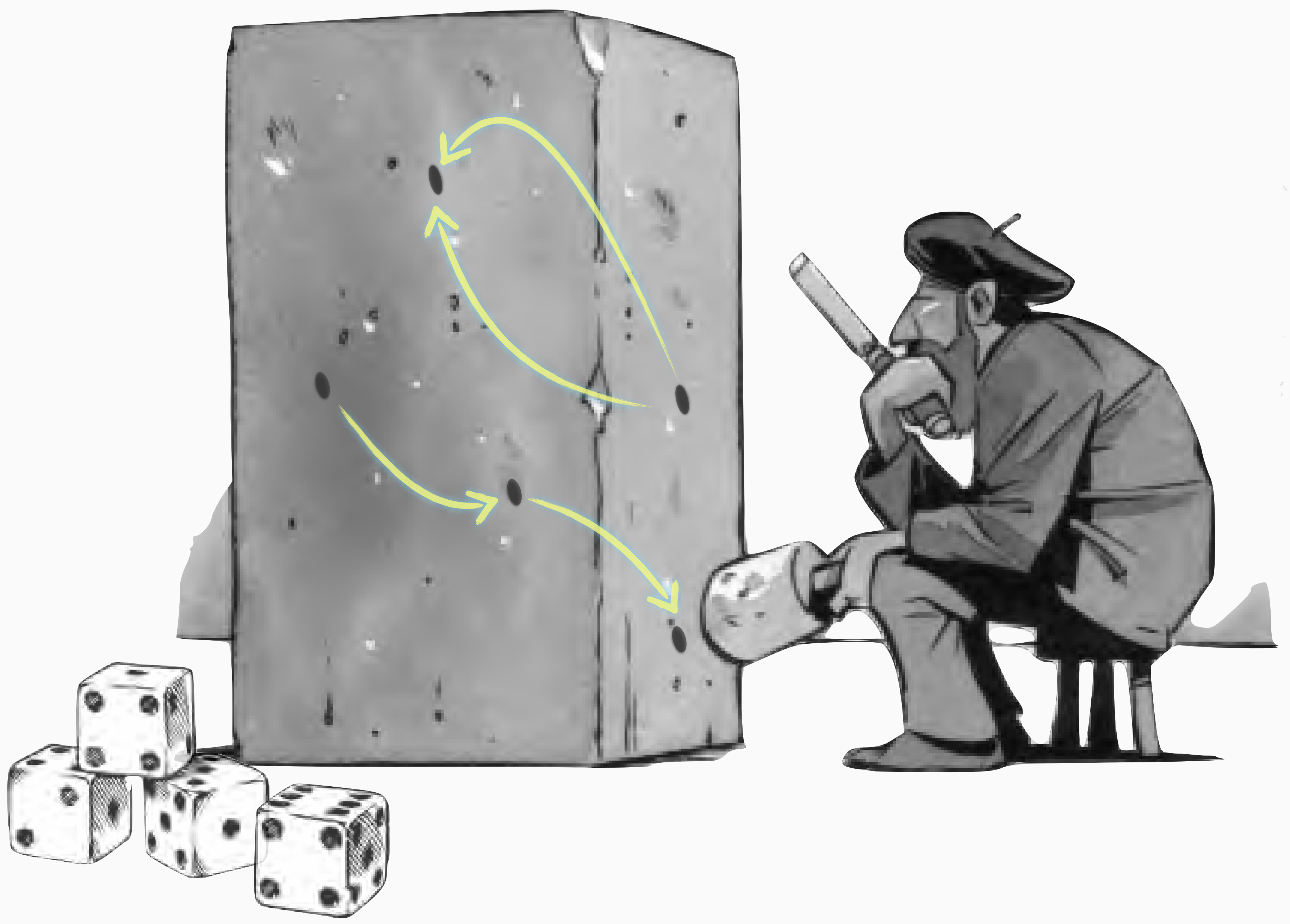
Restricted and well structured set of distributions and their transformations.

*Agent subject to such constraints is **confined** in a very specific world.*





Mapping of classes



Summary

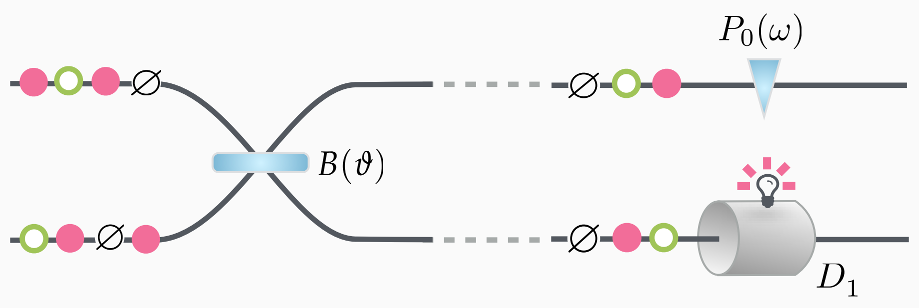
Agent under epistemic constraints

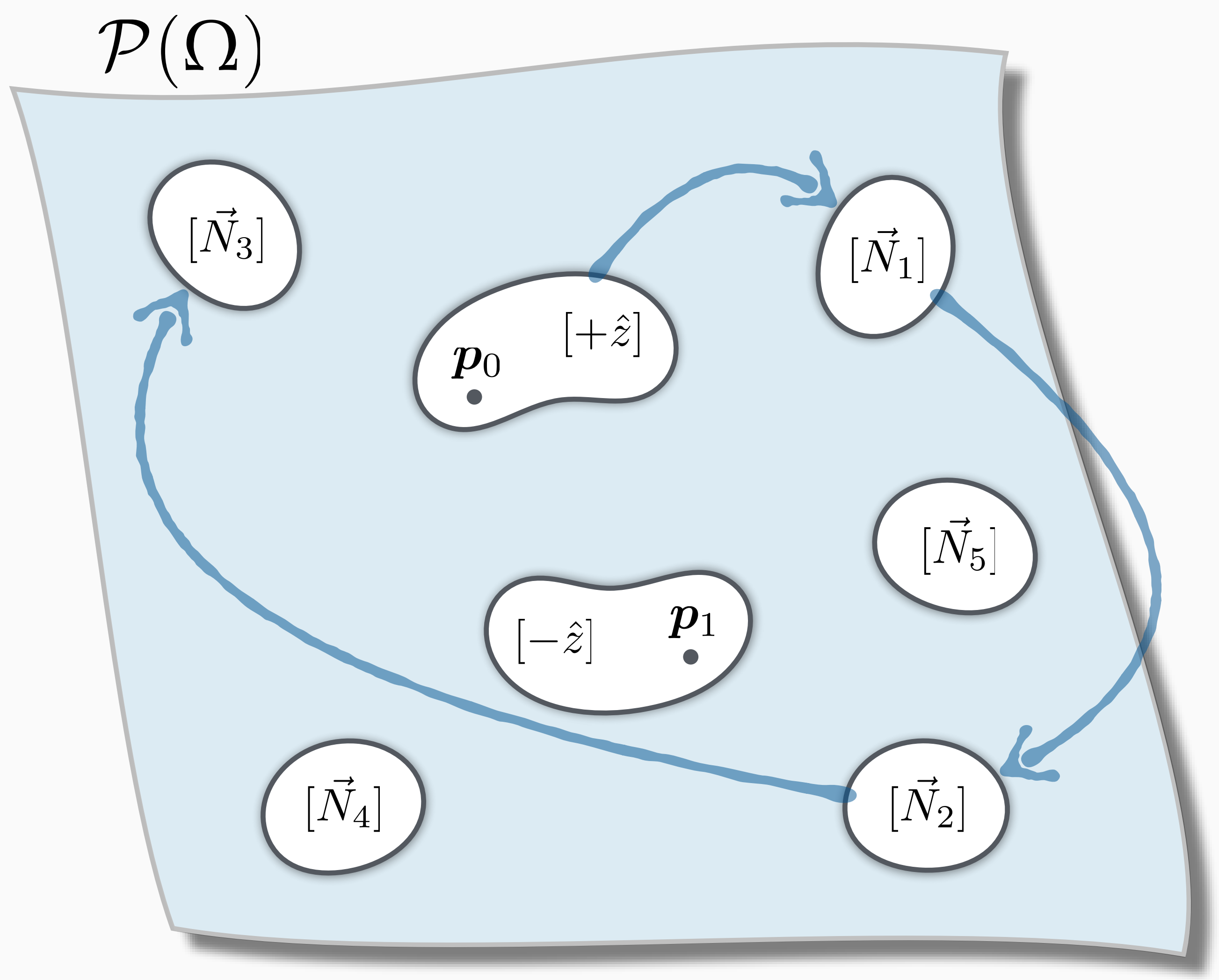
REAL & GHOST particle ontology
*+ **limited** set of **stochastic gates**.*

Restricted and well structured set of distributions and their transformations.

*Agent subject to such constraints is **confined** in a very specific world.*

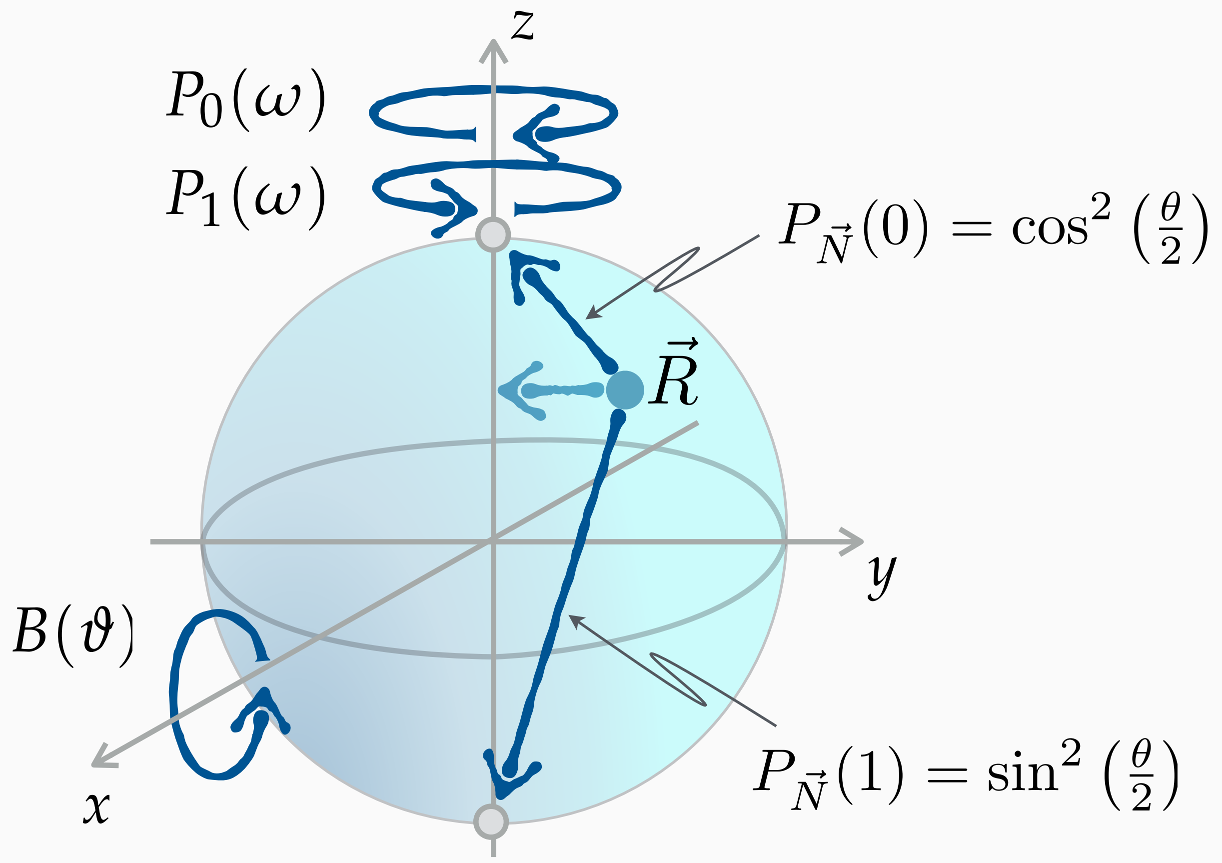
Well-defined local ontology.
Non-locality an epistemic effect.





Mapping of classes

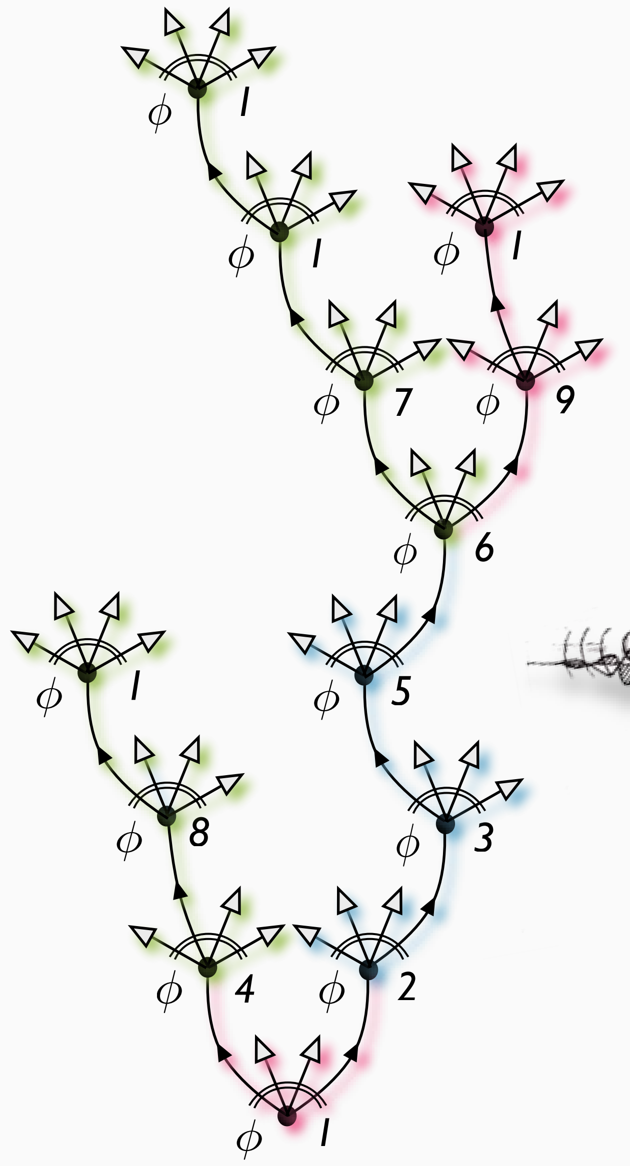
Equivalent

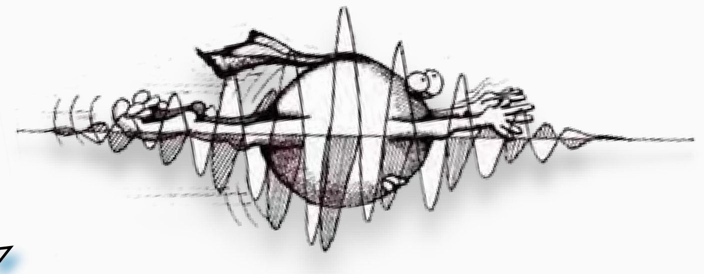


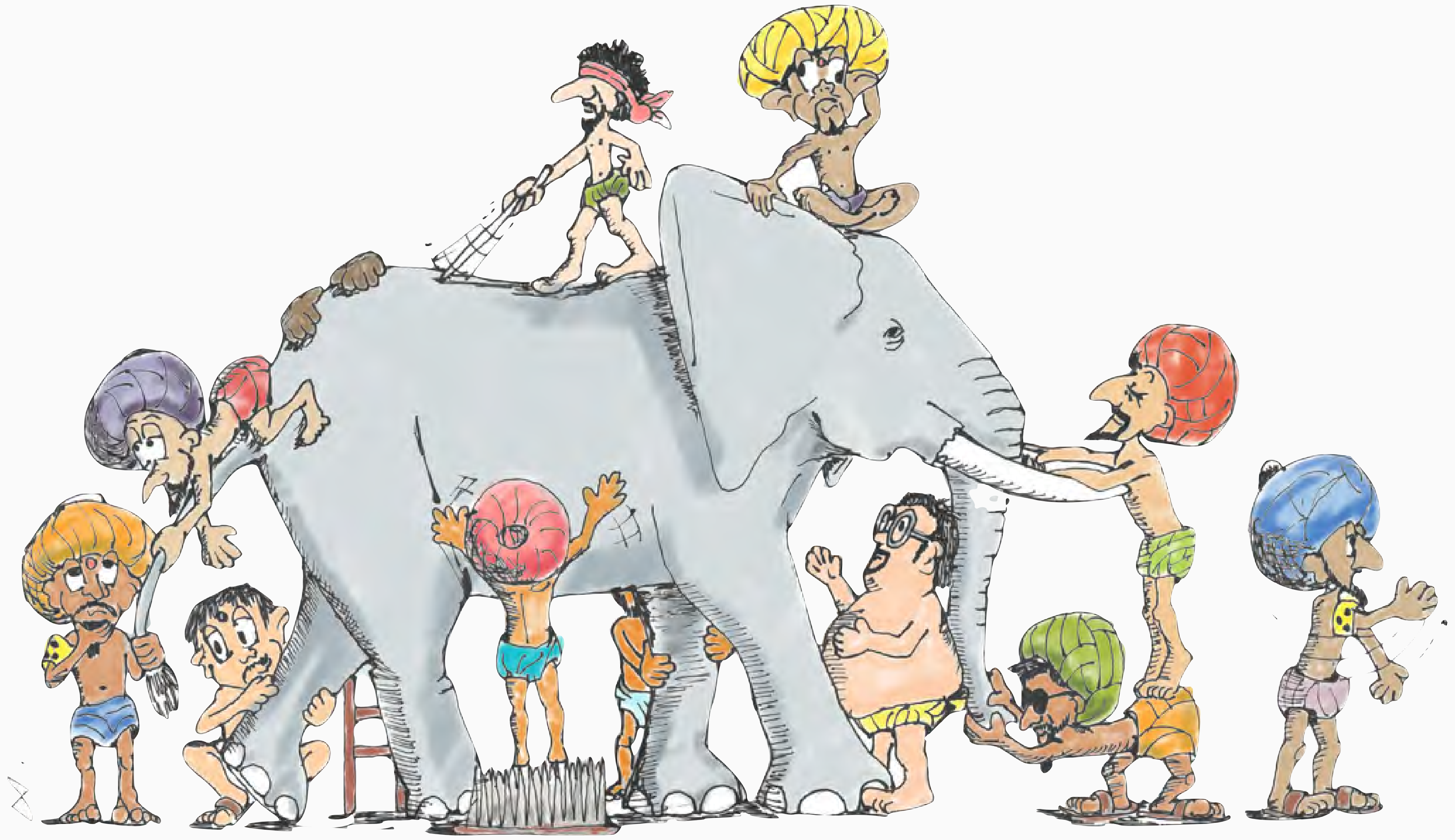
Bloch ball representation

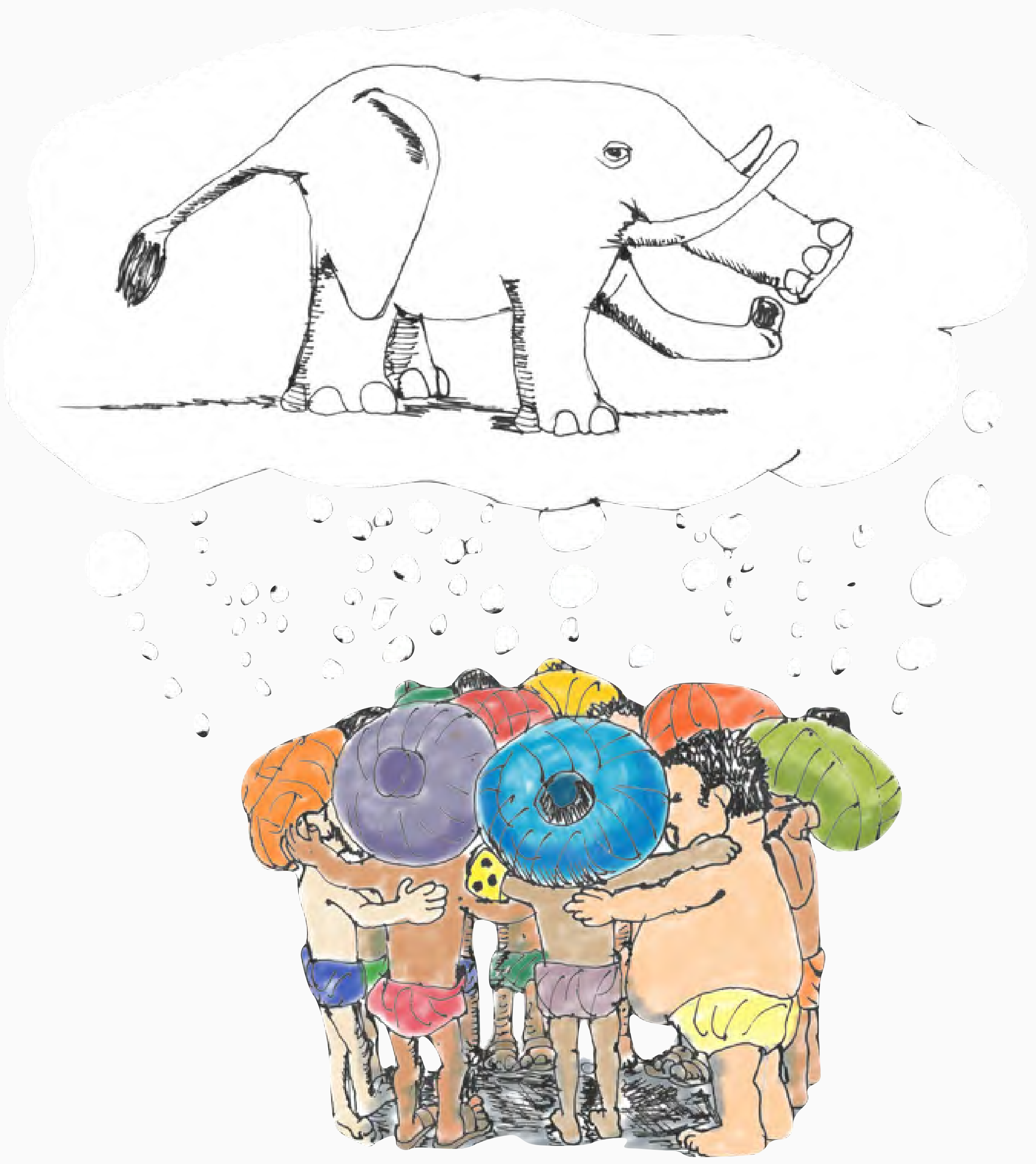
Geometry of accessible states

Thank you









P. Blasiak and P. Flajolet “Combinatorial Models of Creation–Annihilation”
Séminaire Lotharingien de Combinatoire ***65*** *Art. B65c (2011)*

P. Blasiak “Local model of a qubit in the interferometric setup”
New Journal of Physics ***17*** *113043 (2015)*


Urbi et Orbi
Urbi et Orbi